Todays lecture

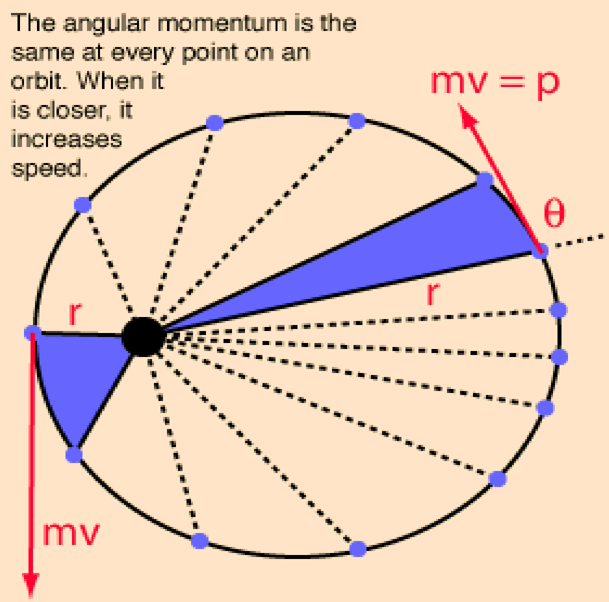
- Refresh basic mechanics
  - Angular momentum
  - Moment of inertia
  - Torque
- Gyros
  - Platforms/compass
  - Strap-down
- Inertial Measurement Units IMU

Material from

- Hyperphysics http://hyperphysics.phy-astr.gsu.edu/
- Fraden Chapter 8
- Bosch/Sensonor
- **Advances in Navigation Sensors and Integration Technology** www.rta.nato.int
- Handbook of Virtual Environment Technology
- InvenSense

Dreie impuls

Angular Momentum of a Particle



The angular momentum of a particle of mass m with respect to a chosen origin is given by

$$L = mvr \sin \theta$$

or more formally by the vector product

$$L = r \times p$$

The direction is given by the right hand rule which would give L the direction out of the diagram. For an orbit, angular momentum is conserved, and this leads to one of Kepler's laws. For a circular orbit, L becomes

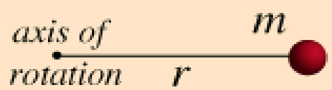
$$L = mvr$$

Dreie impuls og treghetsmoment

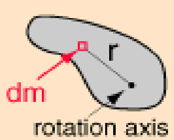
Moment of Inertia, General Form

Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a point mass is defined by

$$I = mr^2$$



then the moment of inertia contribution by an infinitesmal mass element dm has the same form. This kind of mass element is called a differential element of mass and its moment of inertia is given by



$$dI = r^2 dm$$

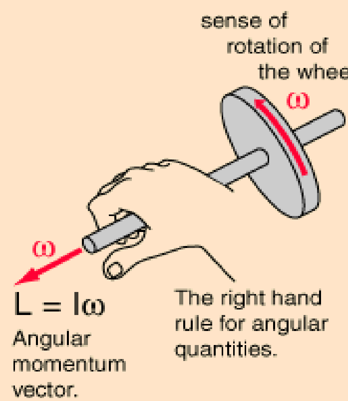
The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

Note that the differential element of moment of inertia dI must always be defined with respect to a specific rotation axis. The sum over all these mass elements is called an integral over the mass.

$$I = \int dI = \int_0^M r^2 dm$$

Angular Momentum

The angular momentum of a rigid object is defined as the product of the moment of inertia and the angular velocity. It is analogous to linear momentum and is subject to the fundamental constraints of the conservation of angular momentum principle if there is no external torque on the object. Angular momentum is a vector quantity. It is derivable from the expression for the angular momentum of a particle



Angular Momentum = Moment of Inertia x Angular Velocity

$$L = I \times \omega$$

Comparison of linear and angular momentum

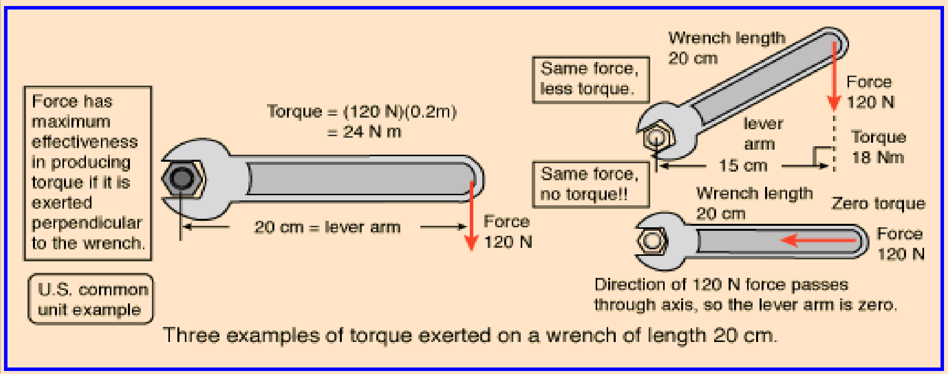
(Kraft-) moment

Torque

A torque is an influence which tends to change the rotational motion of an object. One way to quantify a torque is

Torque = Force applied x lever arm

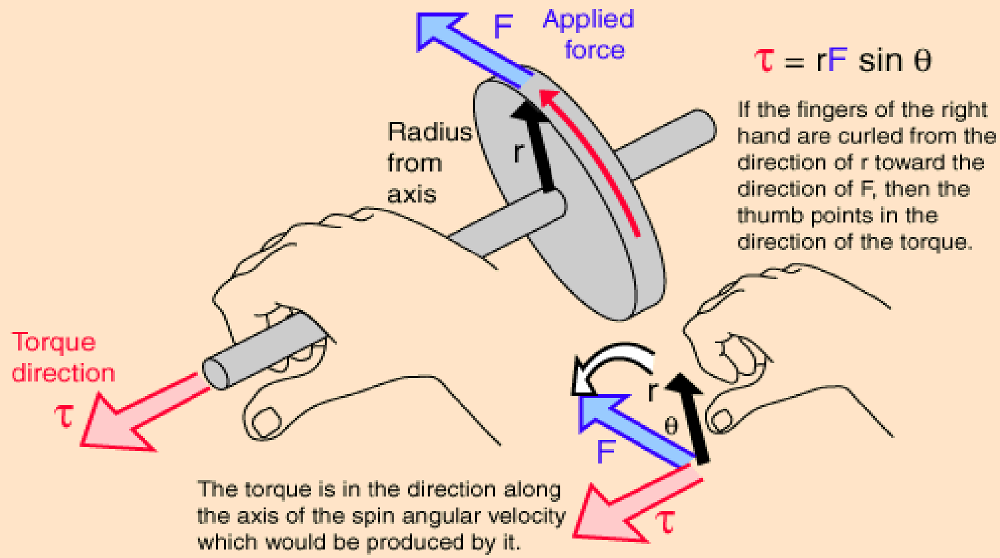
The lever arm is defined as the perpendicular distance from the axis of rotation to the line of action of the force.



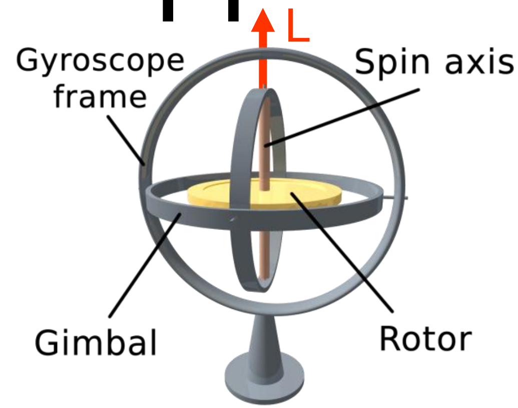
Three examples of torque exerted on a wrench of length 20 cm.

Right Hand Rule for Torque

Torque is inherently a vector quantity. Part of the torque calculation is the determination of direction. The direction is perpendicular to both the radius from the axis and to the force. It is conventional to choose it in the right hand rule direction along the axis of rotation. The torque is in the direction of the angular velocity which would be produced by it in the absence of other influences. In general, the change in angular velocity is in the direction of the torque.



Mekaniske gyro prinsipper



Gyro kompass
(Gyro stabilized platform)

=0

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

Måles

Rate gyro
(Strap-down gyro)

Følger ”bilen”

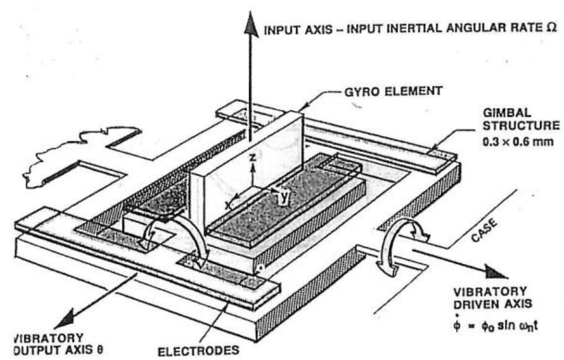


Fig. 8.12. Vibratory rate gyro concept. (From Ref. [7].)

Gyro compass

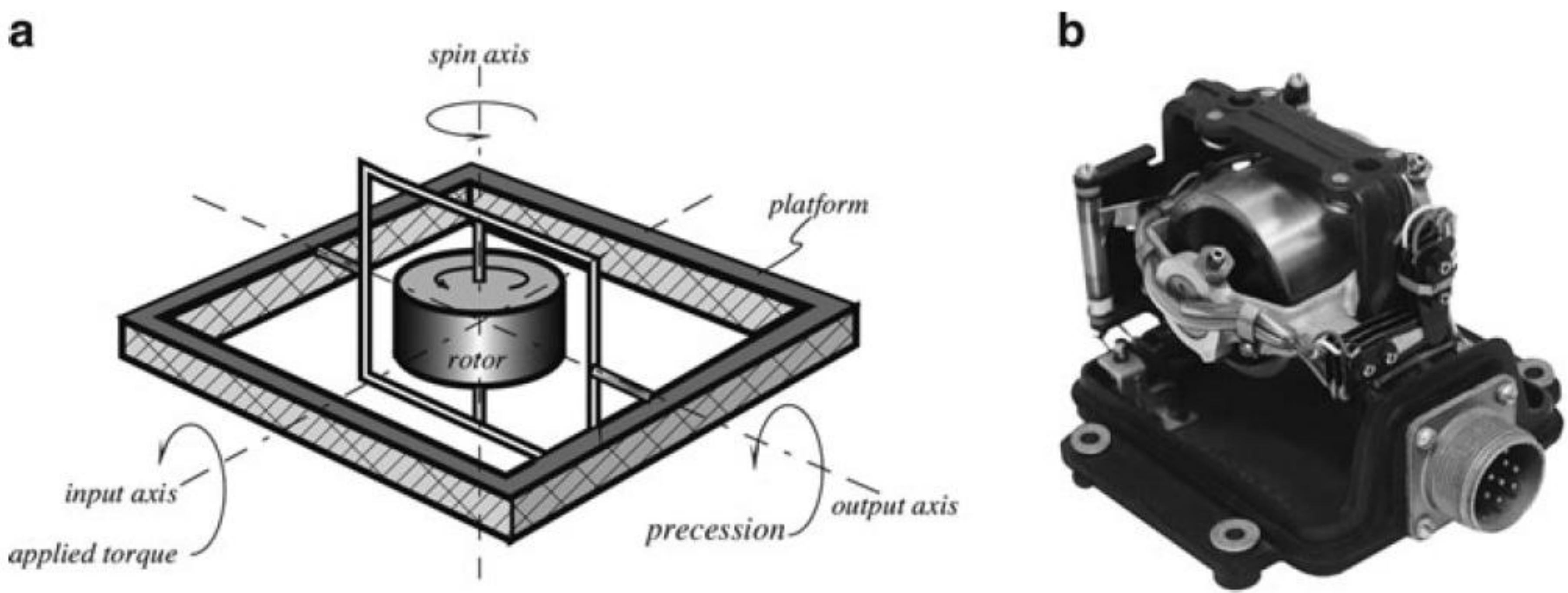
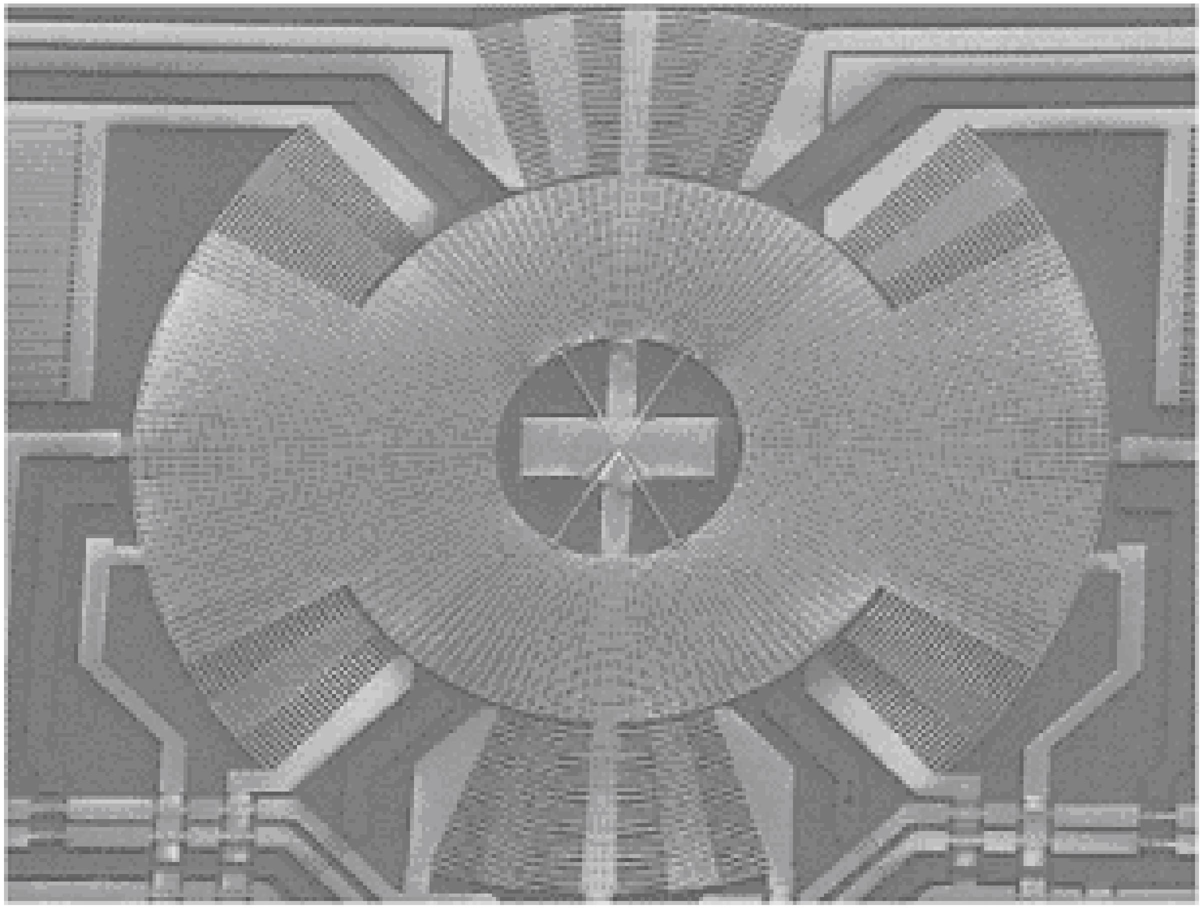
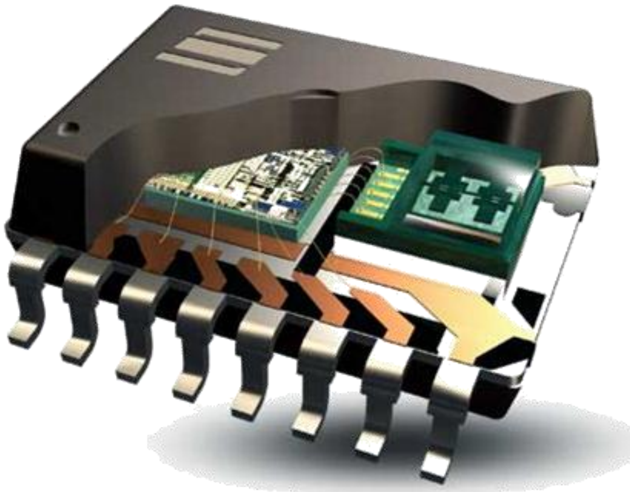


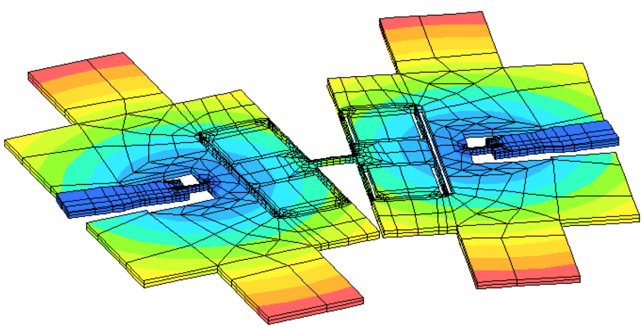
Fig. 8.10 Mechanical gyroscope with a single degree-of-freedom (**a**) and early auto-pilot gyroscope (**b**)

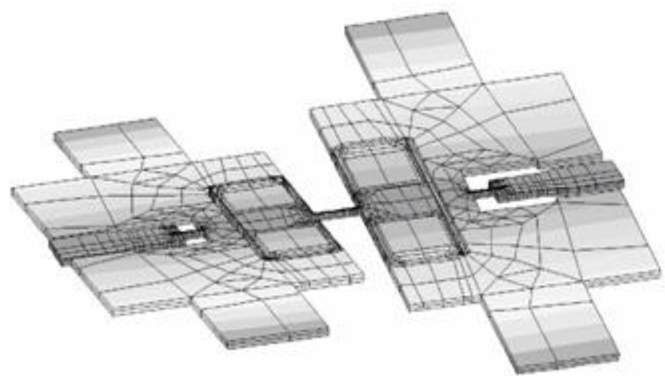
Bosch strap down gyro

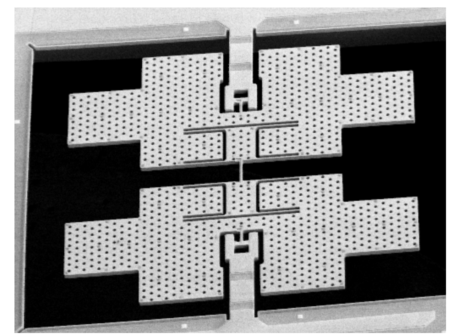


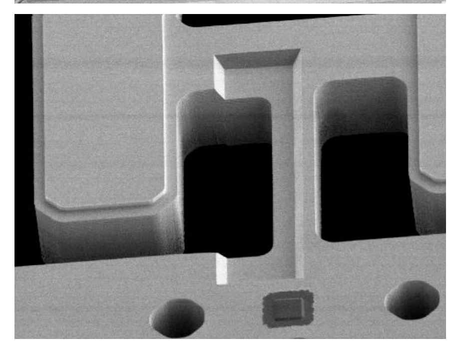
Automotive rate gyro (strap down)





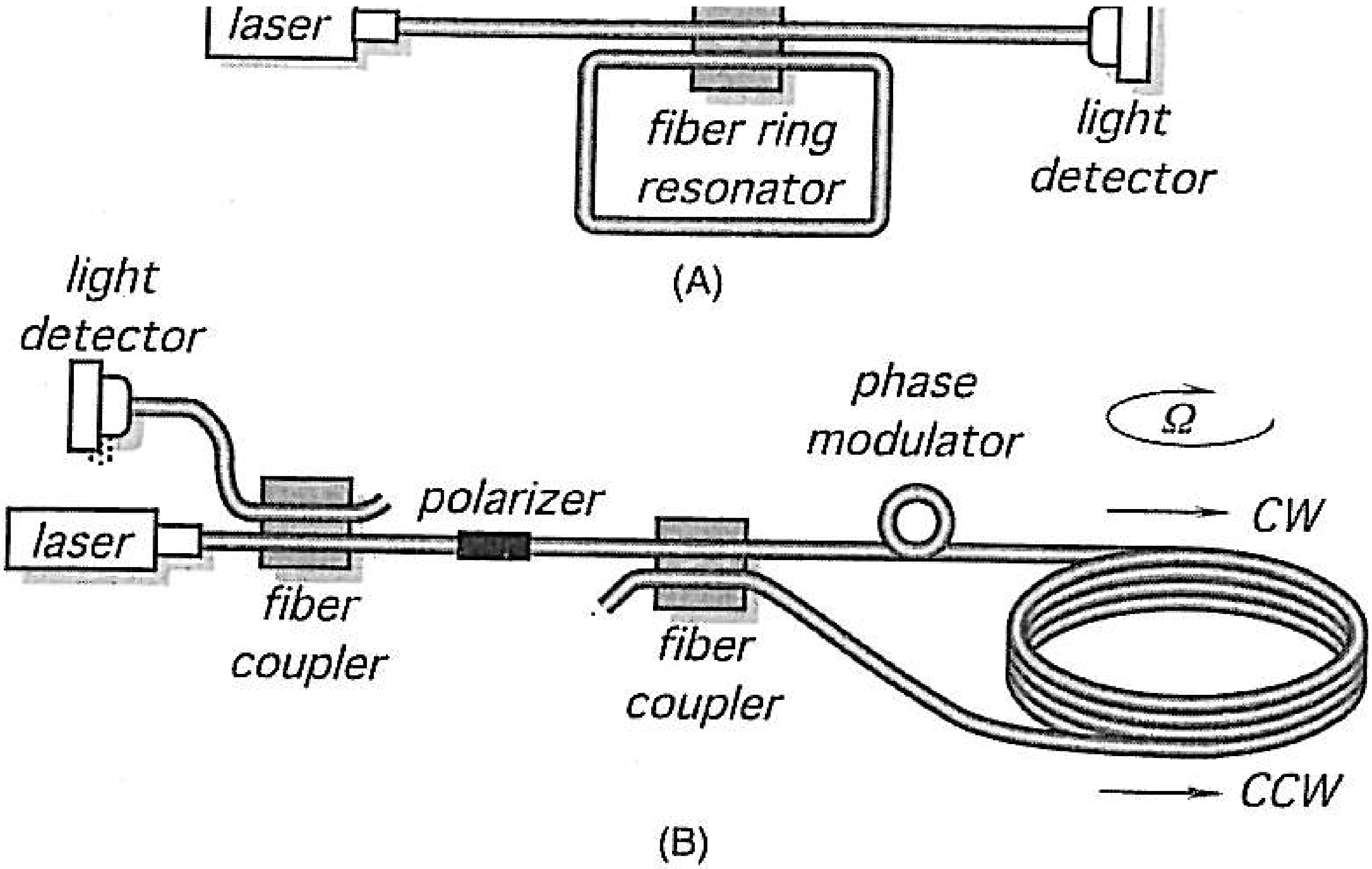






Sensonor SAR 10

Fiber optisk ring gyro



8.14. (A) Fiber-optic ring resonator; (B) fiber-optic analog coil gyro. (Adapted fr
[9].)

IMU-INS

- Inertial measurement unit (IMU)
- Inertial navigation system (INS)

Figure 1. SPIRE system.

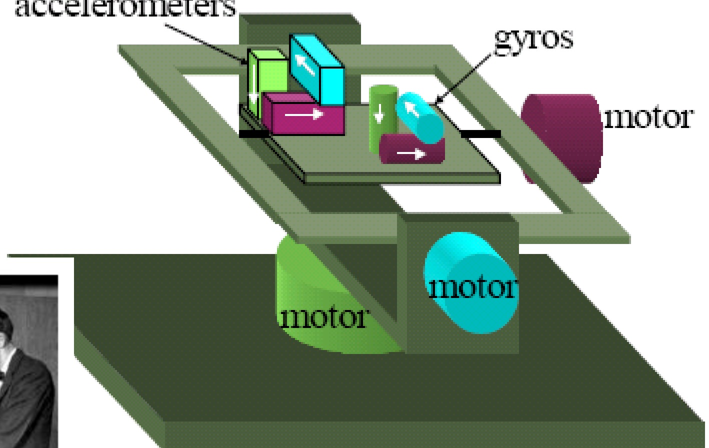


Figure 2a : Stable-platform INS

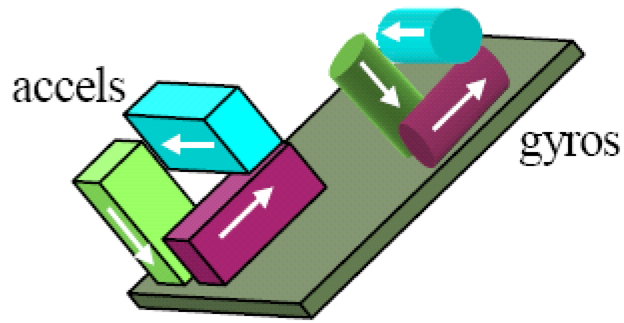


Figure 2b: Strapdown I

Militære anvendelser og krav

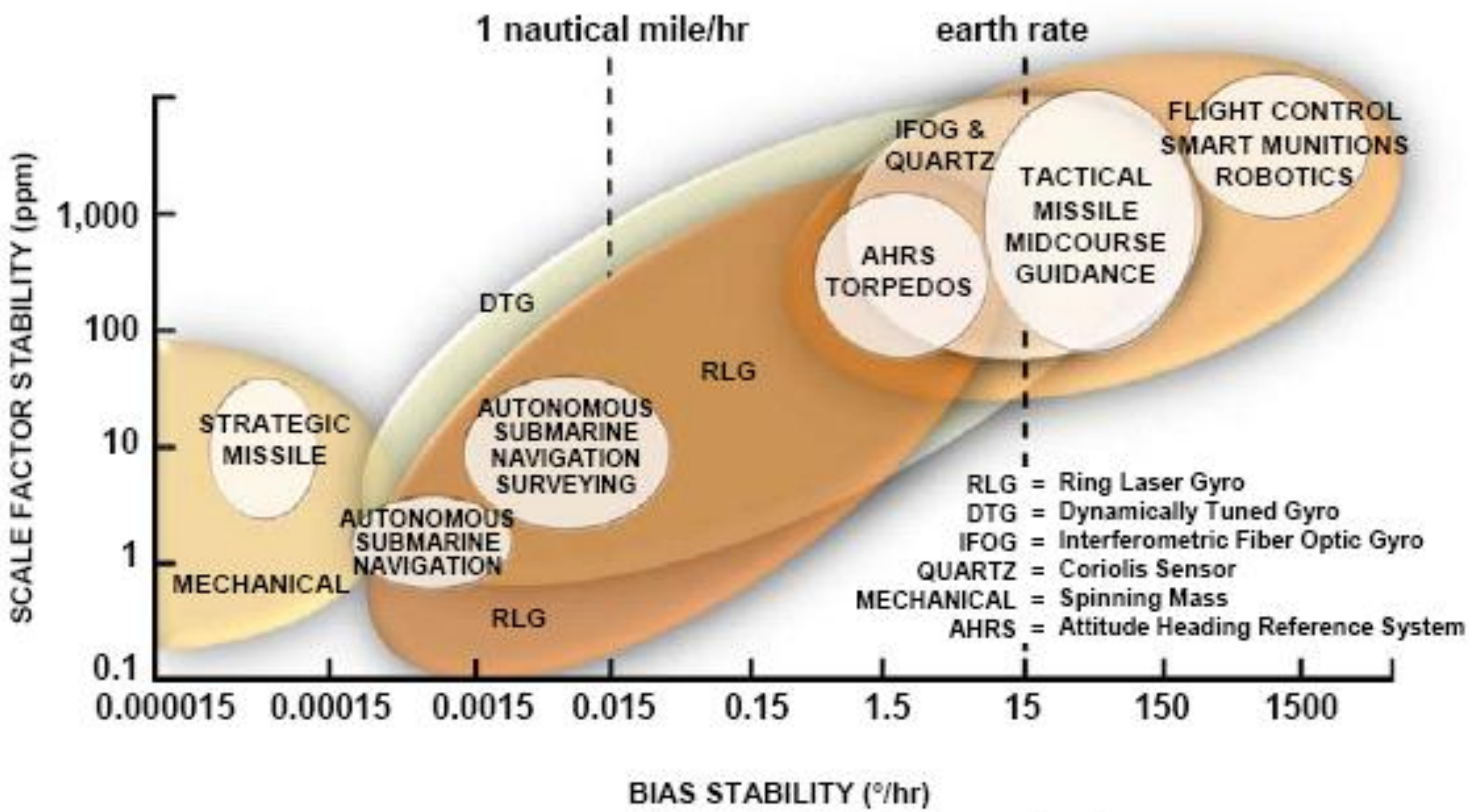


Figure 3. Current gyro technology applications.

	Commercial-Grade	Tactical-grade	Navigation-grade	Strategic-grade	Geophysical limit
Gyro bias stability	1500°/hr/√hr	15°/hr/√hr	0.015°/hr/√hr	0.000015°/hr/√hr	0°/ hr/√hr
Gyro bias initial uncertainty	150°/hr	1.5°/hr	0.0015°/hr	0.0000015°/hr	0°/hr
Accel bias stability	1 mg/√hr	100 μg/√hr	10 μg/√hr	0.5 μg/√hr	0 μg/√hr
Accel bias initial uncertainty	0.25 mg	10 μg	1 μg	0.1 μg	0.1 μg
Initial orientation alignment	1 arcsecond	1 arcsecond	1 arcsecond	0.1 μrad	0.01 μrad

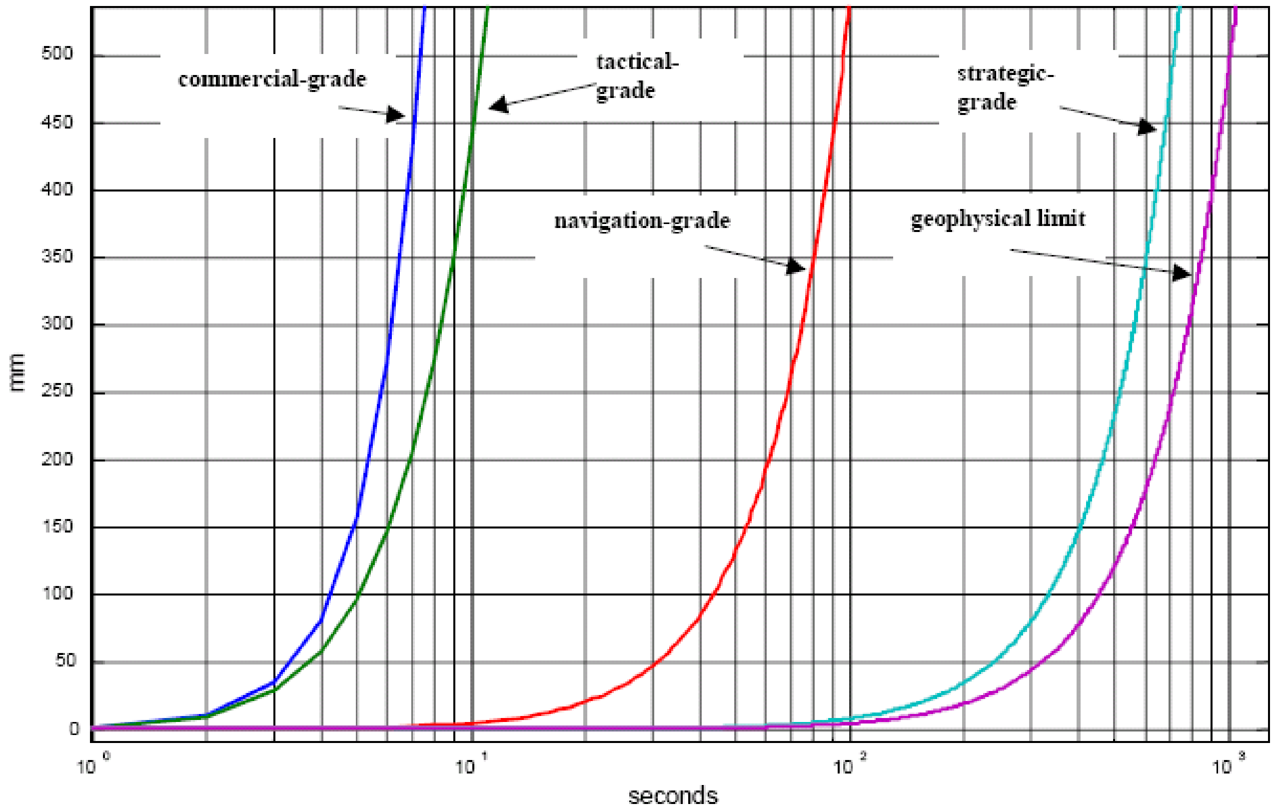
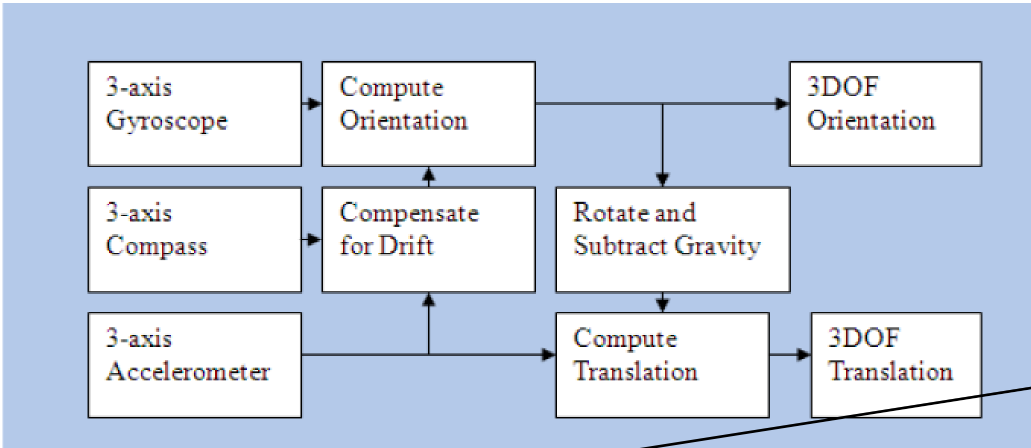


Figure 4: Comparison of 1-σ random position drift performance of commercial, tactical, navigation, strategic-grade, and "perfect" inertial navigation systems over a 20 minute covariance simulation.

Handbook of Virtual Environment Technology

Smart phone IMU



+ Trykk

Figure 9. 9-axis sensor fusion algorithm. Information from gyroscope, accelerometer, and digital compass are integrated to generate 6 degrees-of-freedom (DOF) motion information (3-axis orientation and 3-axis translation).

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