*FYS3500 - spring 2019*

# 2-Generation Neutrino-Mixing*

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***Martin and Shaw, Particle Physics, 4th Ed., Section 2.3 (Last update 18.02.2018 16:36)**

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- This is interpreted as the flavor eigenstates not being synonymous with mass-eigenstates, but rather mixtures of mass eigenstates that propagate differently due to mass differences.
- The neutrinos propagate as mass eigenstates, however, they are produced and detected as flavor eigenstates of the weak interactions (via $W^\pm$ and $Z^0$ bosons).

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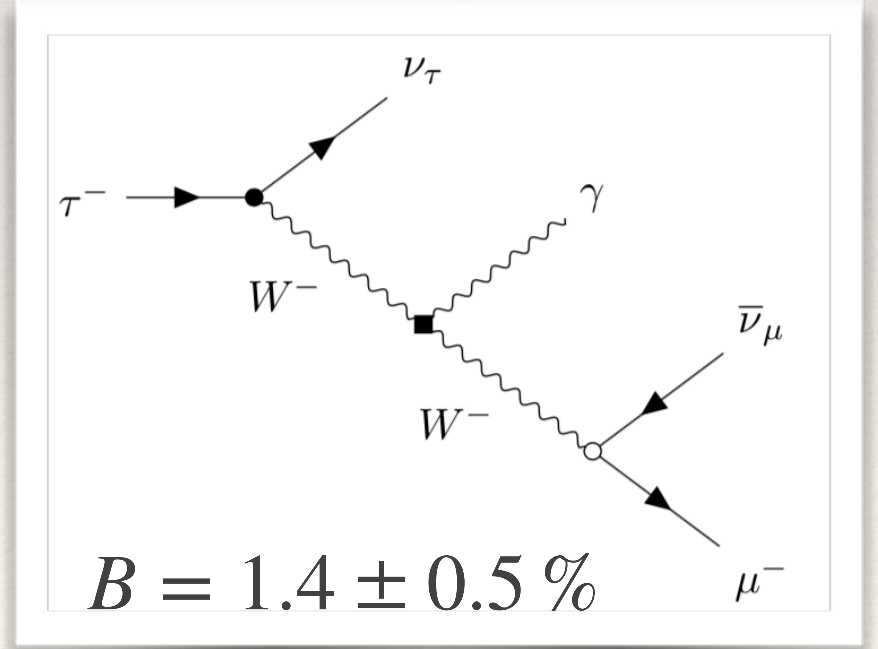

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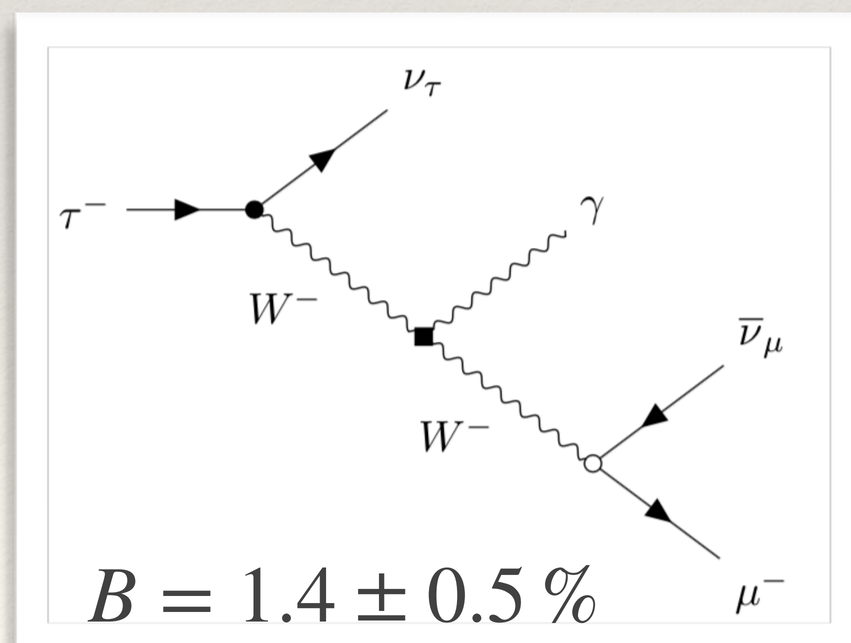


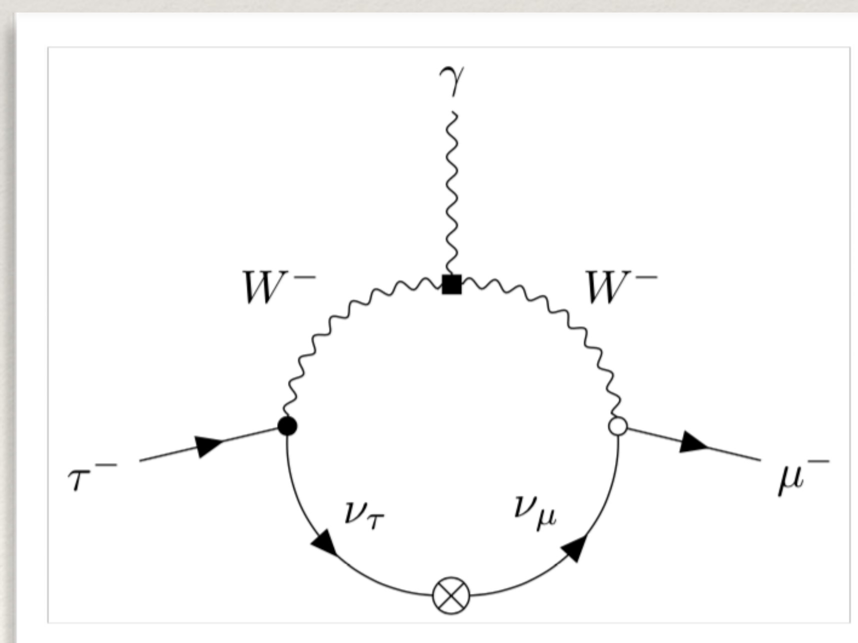

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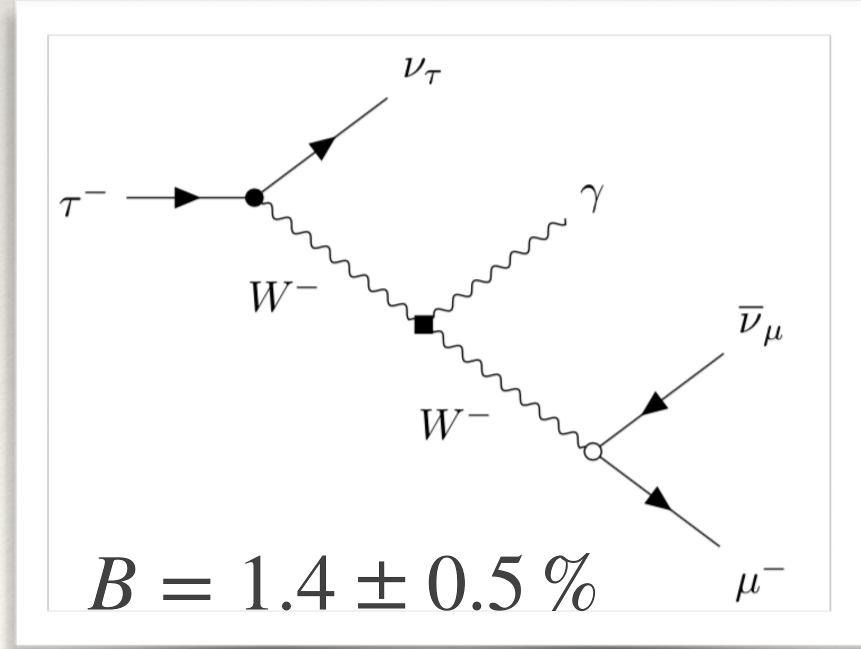
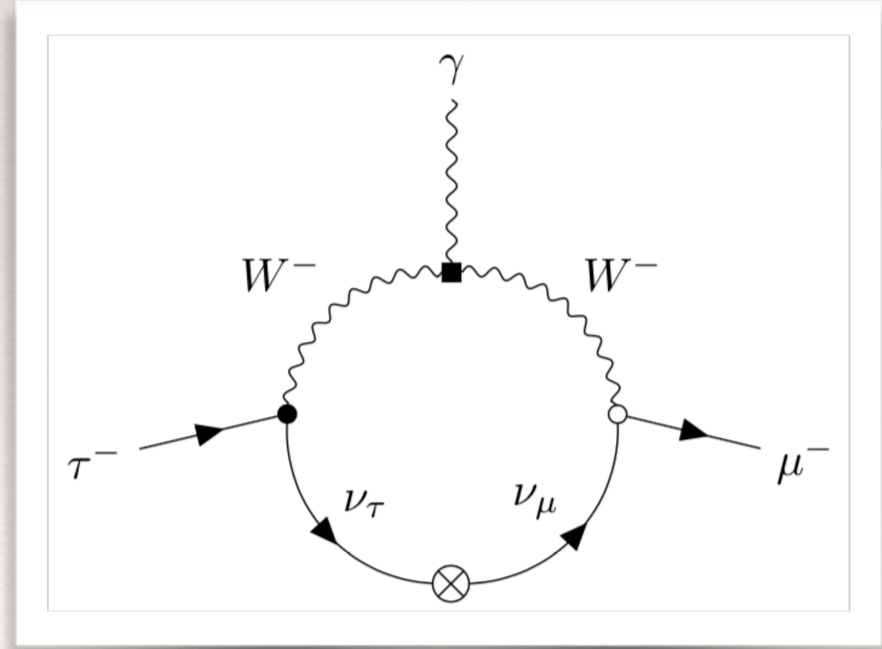
$$B(\tau^- \to \mu^- \gamma) = O(B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) \cdot \alpha_{EM} \cdot P(\nu_\tau \to \nu_\mu))$$
$$= O(17\,\% \cdot \sim 1\,\% \cdot \ll 1)$$



# 3-generation mixing

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- We can understand many features of 3-generation mixing by studying the simpler case with only 2 generations.
- Imagine producing a beam of electron neutrinos with a specified momentum $p$ and observing them a time later at some distance $x$.
- Since the neutrinos are produced with a mix of mass eigenstates, and since the mass eigenstates propagate (slightly) differently due to their mass difference, some of the electron neutrinos will transform to muon neutrinos.

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- To simplify the notation I will use $\theta = \theta_{12}$ to indicate the mixing between the two mass eigenstates.

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- For $t \neq 0$ the mixture of mass eigenstates will have changed, leading to the disappearance of $\nu_e$ and the appearance of $\nu_\mu$.

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- Recall that $|<\nu_e|\nu_e>|^2 = |<\nu_\mu|\nu_\mu>|^2 = 1$

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$$\left|b(t)-a(t)\right|^2 = b^*b + a^*a - (a*b + b*a)$$

$$= 2 - e^{i(E_2-E_1)t} - e^{-i(E_2-E_1)t}$$

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$$P(\nu_e \to \nu_\mu) = |<\nu_\mu|\nu(t)>|^2 = \sin^2\theta \sin^2\left(\frac{L}{L_0}\right)$$

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- Negligible impact on weak interactions due to distance scale O($10^{-15}$) m

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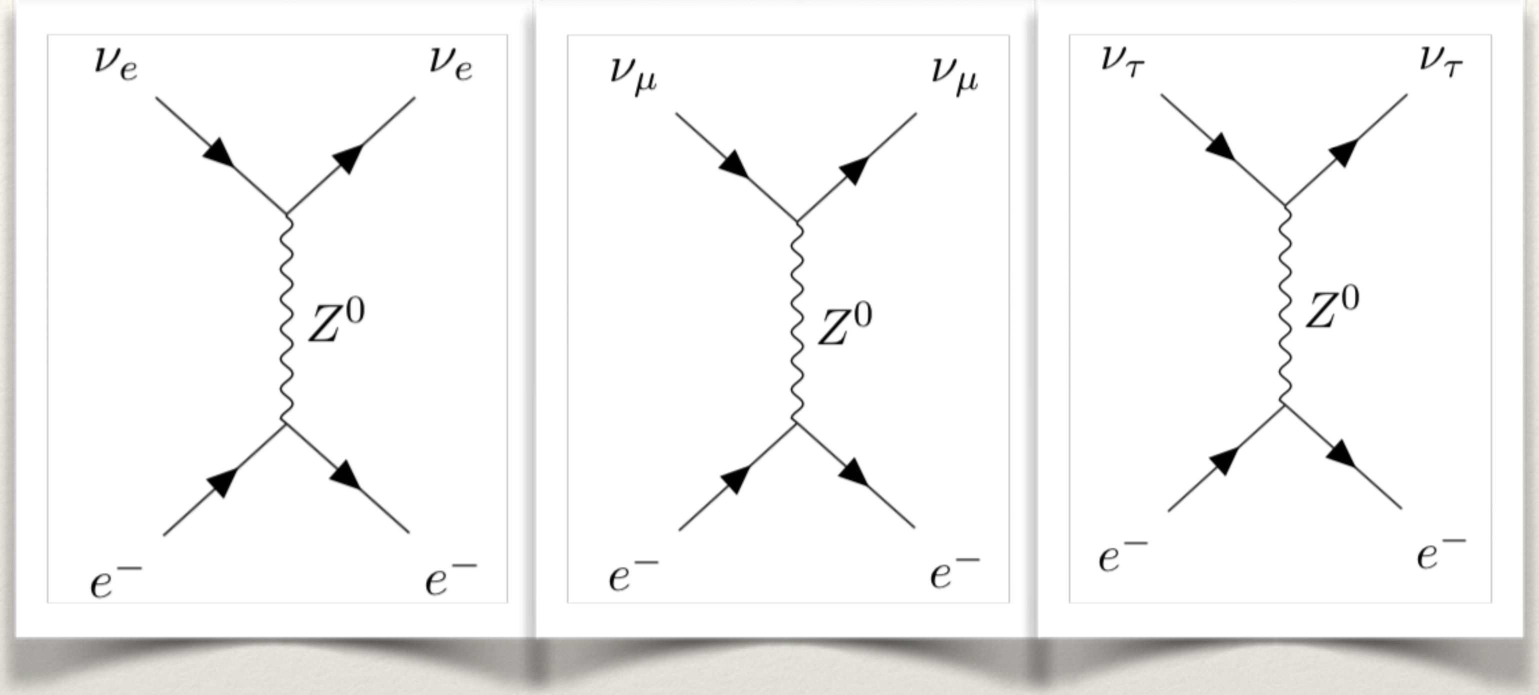
- There are (at least) 3 mass eigenstates
  - Mixing between 1,2 is large
  - Mixing to 3 is small
  - Neutrino masses and differences below the eV scale, consistent with calculated impact on cosmology
  - A few results somewhat easier to understand if there is a 4th “sterile” neutrino that mixes with the others but doesn’t interact with other SM particles

# Neutrino mixing in practice - II

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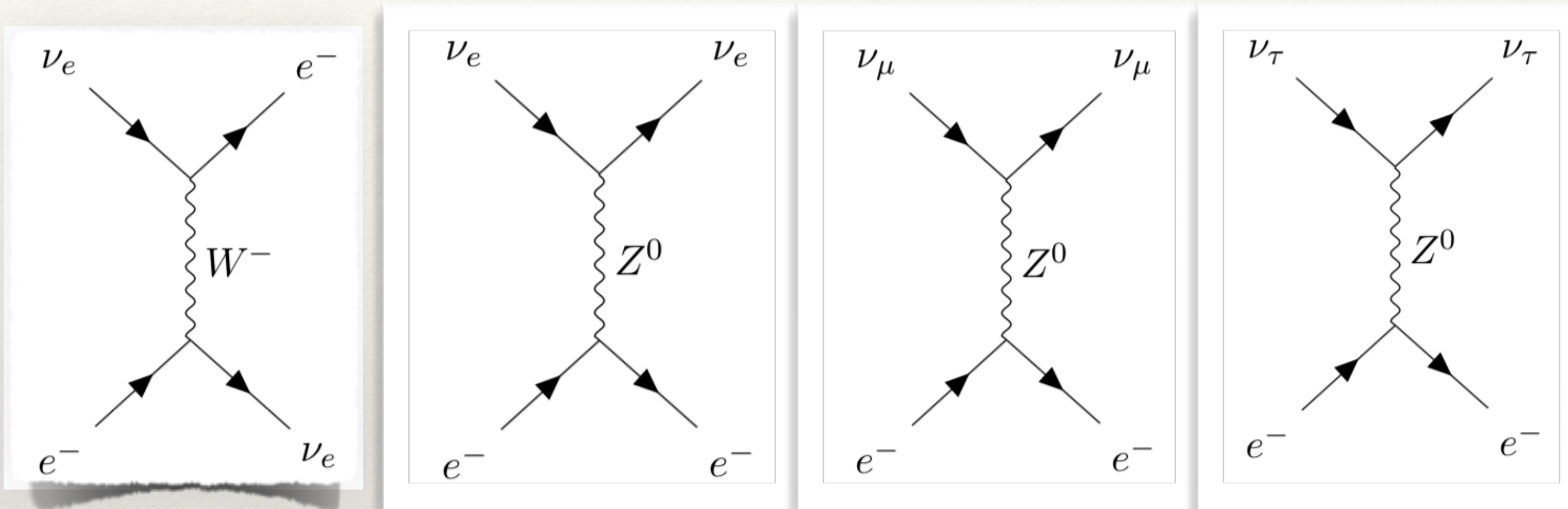


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