

*FYS3500 - spring 2019*

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# Quark Model and Color\*

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Department of Physics

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\*Martin and Shaw, Particle Physics, 4th Ed., Chapter 6

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# Isospin

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- ❖  $u$  and  $d$  quark masses are similar but not equal, electric charges are not the same and EM interaction is smaller than strong but not zero  $\Rightarrow$  Isospin an approximate symmetry

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- ❖ In practice there are small mass differences

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$$I_3 = \sum_i I_3^i, \quad \text{where } I_3(u, \bar{d}) = \frac{1}{2}, \quad I_3(\bar{u}, d) = -\frac{1}{2}$$

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$$I_3 = \frac{1}{2}(N_u - N_d) - \frac{1}{2}(N_{\bar{u}} - N_{\bar{d}})$$

- ❖ Can show that (in fact this came before quarks)

$$I_3 = Q - \frac{Y}{2}$$

# $Y, I$ Quantum numbers (quarks)

$$Y = B + S + C + \tilde{B} + T$$

Quark	$B$	$Y$	$Q$	$I$	$I_3$
$u$	$1/3$	$1/3$	$2/3$	$1/2$	$1/2$
$d$	$1/3$	$1/3$	$-1/3$	$1/2$	$-1/2$
$c$	$1/3$	$4/3$	$2/3$	$0$	$0$
$s$	$1/3$	$-2/3$	$-1/3$	$0$	$0$
$t$	$1/3$	$4/3$	$2/3$	$0$	$0$
$b$	$1/3$	$-2/3$	$-1/3$	$0$	$0$

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$\bar{u}$	$-1/3$	$-1/3$	$-2/3$	$1/2$	$-1/2$
$c$	$1/3$	$4/3$	$2/3$	$0$	$0$
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$b$	$1/3$	$-2/3$	$-1/3$	$0$	$0$

- ❖ For antiquarks  $I$  is same but the rest change sign

# Hadrons with $C = \tilde{B} = T = 0$

Isospin states:  $\frac{1}{2} + \frac{1}{2} = (0,1)$        $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = (0,1) + \frac{1}{2} = \frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

Hadrons	Quarks	$S$	$I$
<i>Baryons</i>	$qqq$	0	$3/2, 1/2$
	$qq_s$	-1	1, 0
	$q_s s$	-2	$1/2$
	$sss$	-3	0
<i>Mesons</i>	$q\bar{s}$	1	$1/2$
	$s\bar{s}$	0	0
	$q\bar{q}$	0	1, 0
	$\bar{q}s$	-1	$1/2$

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$$K^- + p \rightarrow \pi^- + \Sigma^+$$

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- ❖ “Easy” to produce  $\Rightarrow$  strong interaction
- ❖ Must be a strange baryon with zero hypercharge:

$$B = 1, S = -1 \Rightarrow Y = B + S = 0$$

$$I_3 = Q - Y/2 = 1 - 0 = 1$$

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- ❖ Should belong to  $I=1$  multiplet with 3 particles

$$I_3 = 1, 0, -1 = Q \Rightarrow \Sigma^+, \Sigma^0, \Sigma^-$$

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- ❖ No doubly charged states observed, as expected in the quark model

$$\Sigma^{++}, \Sigma^{--} \quad \times$$

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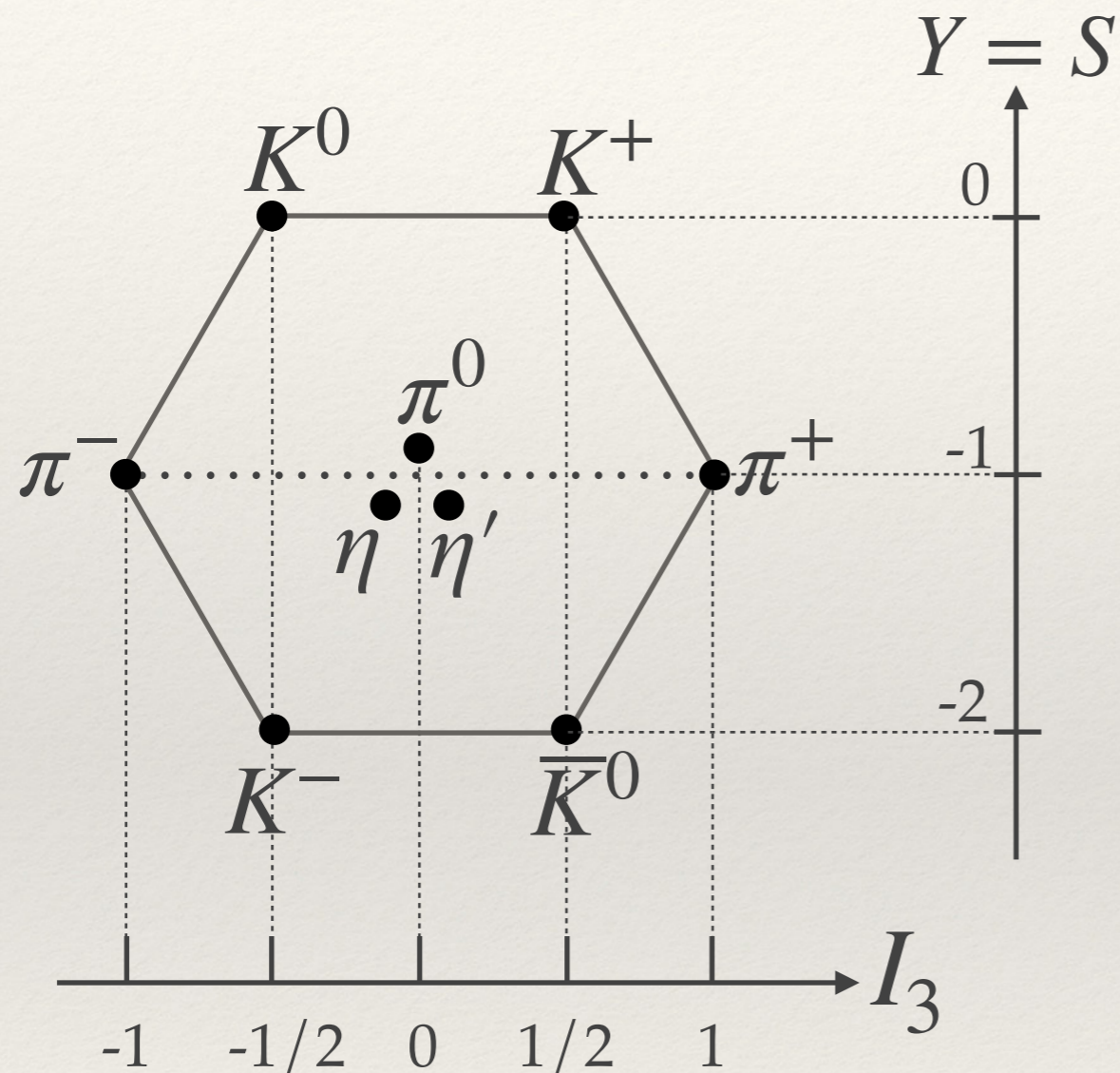
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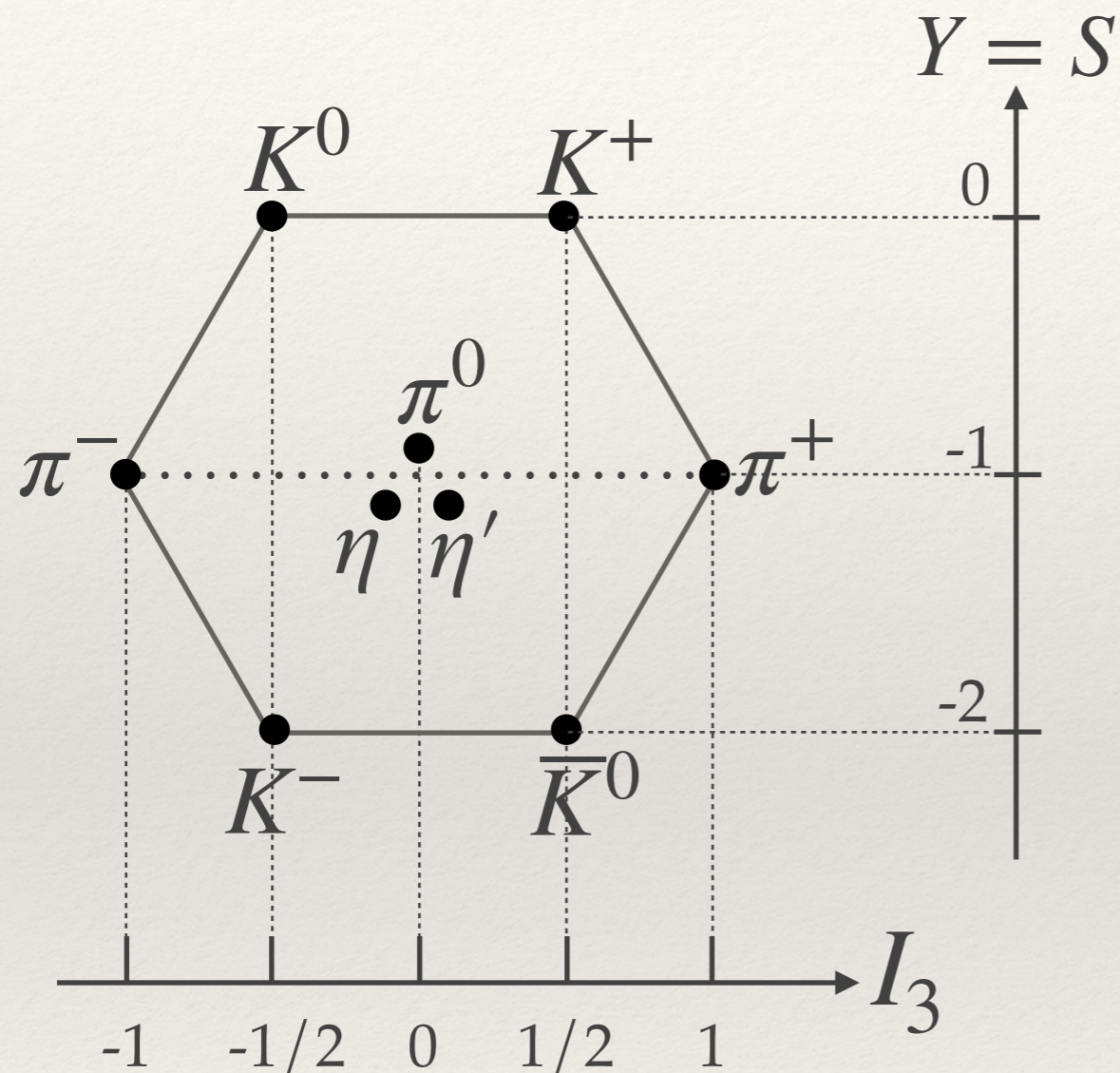
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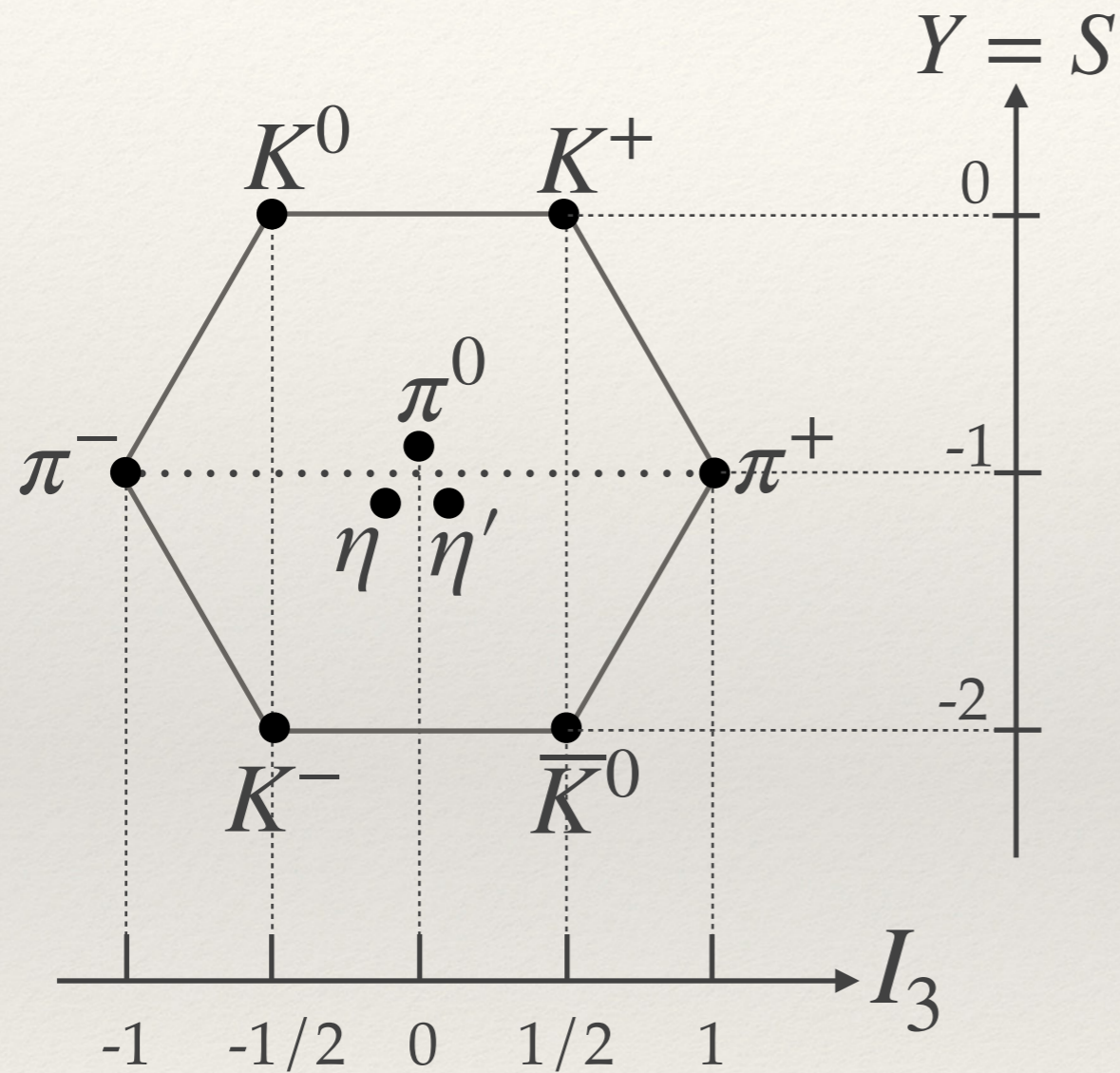


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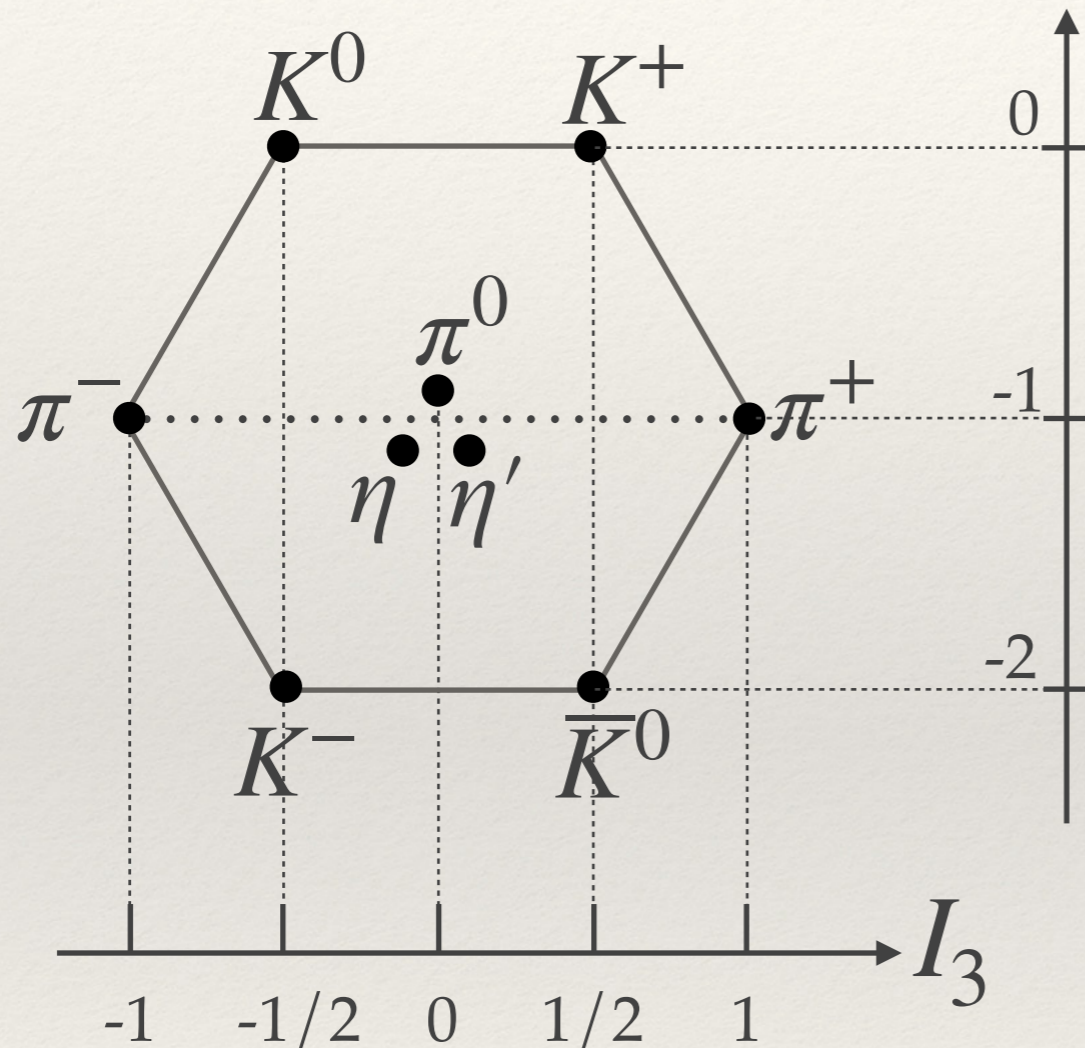


$$J = L + S = 0 + S = \frac{1}{2} + \frac{1}{2} = 0, 1 \quad P = P_q P_{\bar{q}} (-1)^{L=0} = -1 \cdot 1 = -1$$



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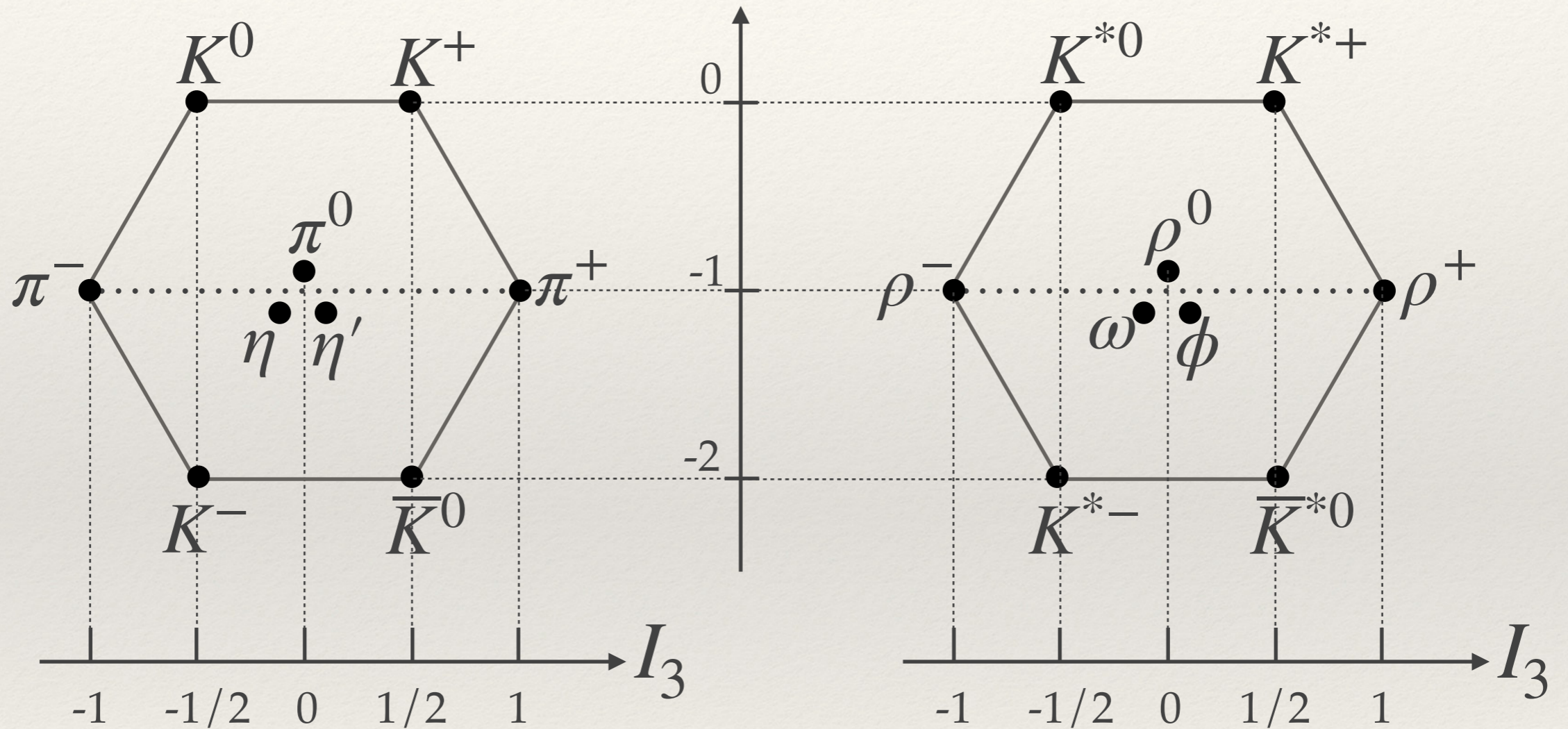
$J^P = 0^-$  "Pseudoscalars"  $Y = S$



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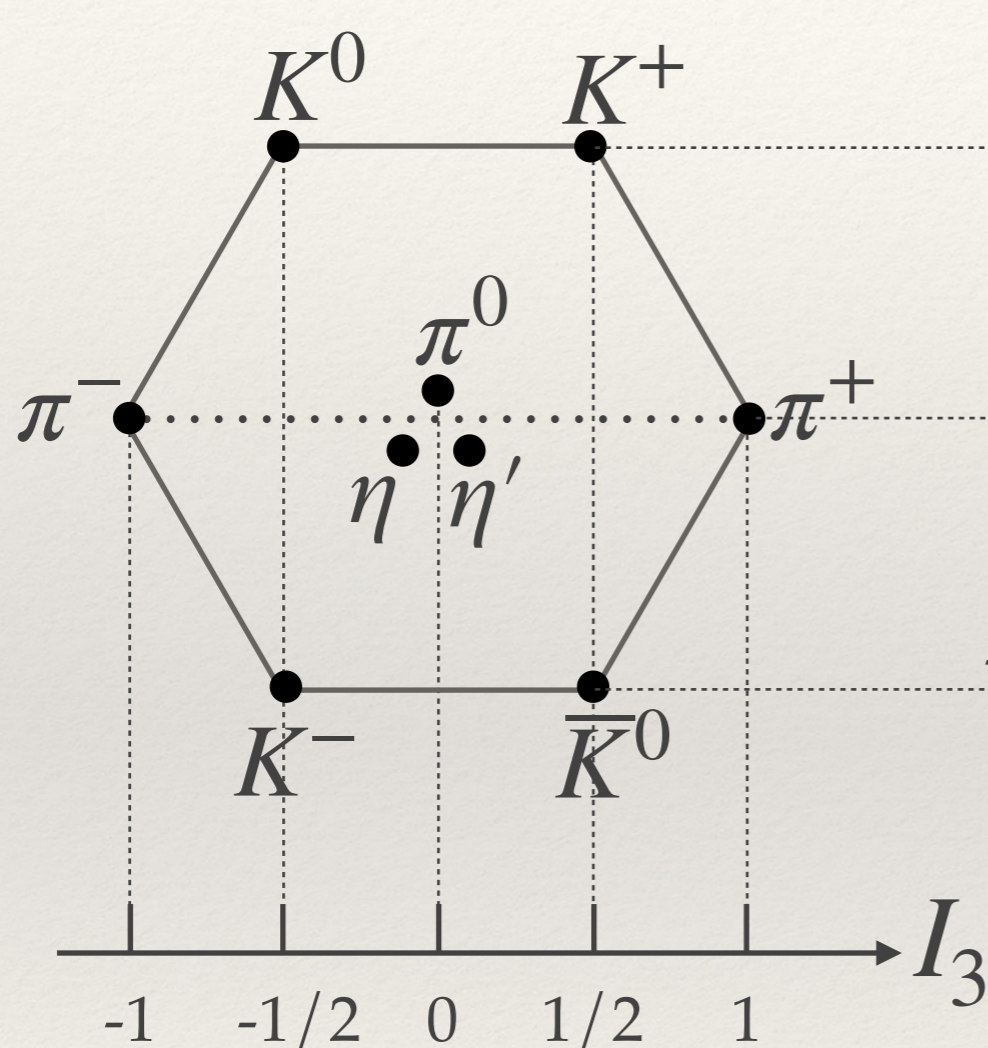
$J^P = 0^-$  "Pseudoscalars"  $Y = S$



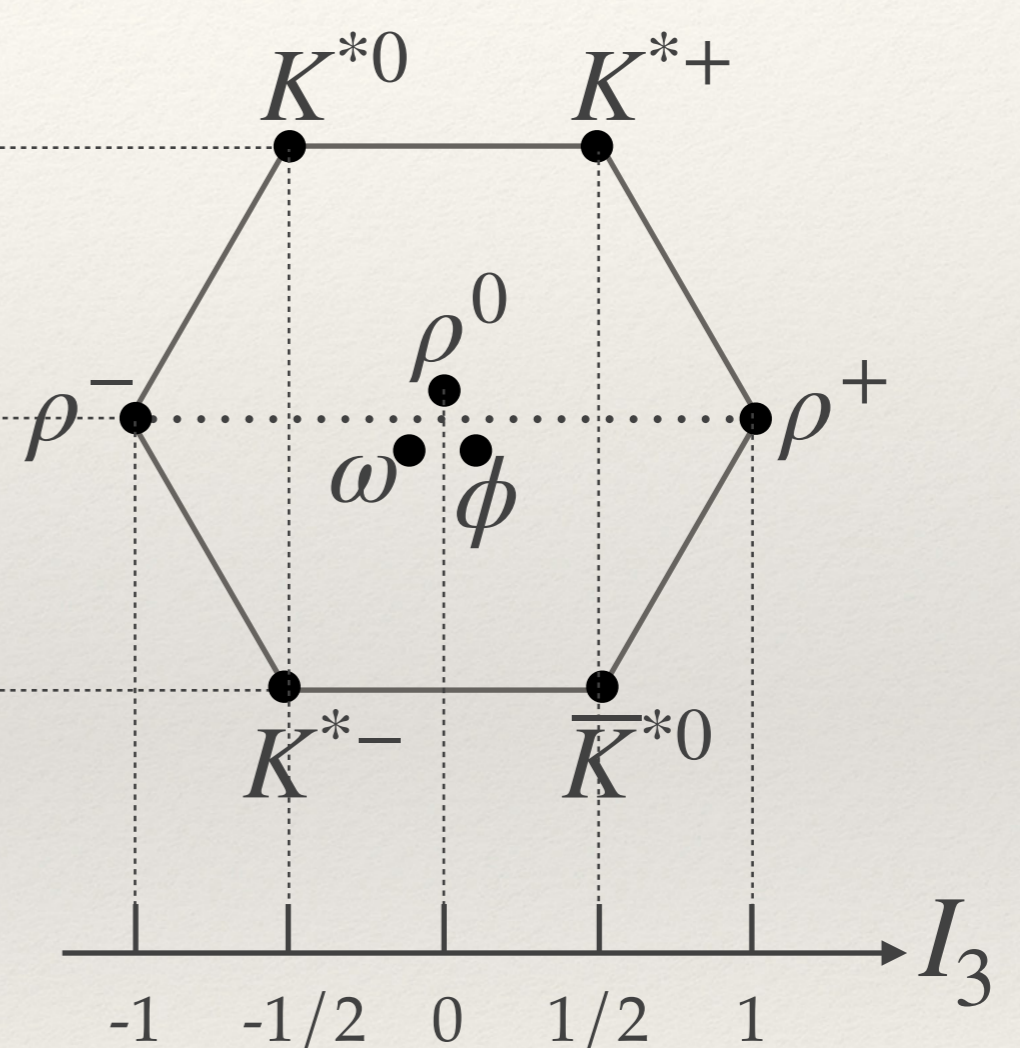
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$J^P = 1^-$  "Vectors"



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Quark content	$J^P=0^-$	Mass	$J^P=1^-$	Mass
$u\bar{d}$	$\pi^+$	140	$\rho^+$	768
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$d\bar{u}$	$\pi^-$	140	$\rho^-$	768
$u\bar{s}$	$K^+$	494	$K^{*+}$	892
$d\bar{s}$	$K^0$	498	$K^{*0}$	896
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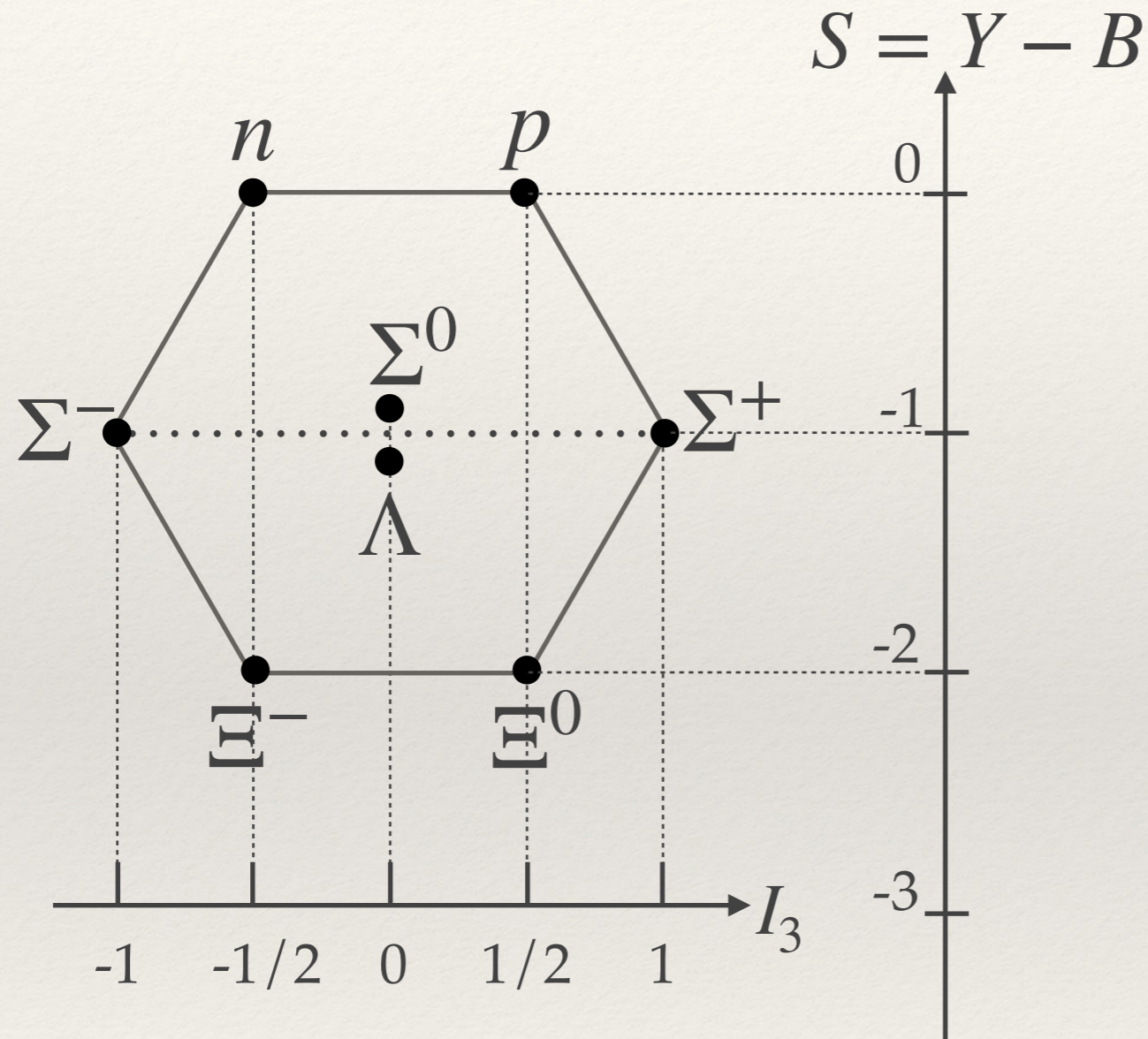
# Light baryon multiplets ( $L_{12}=L_3=0$ )

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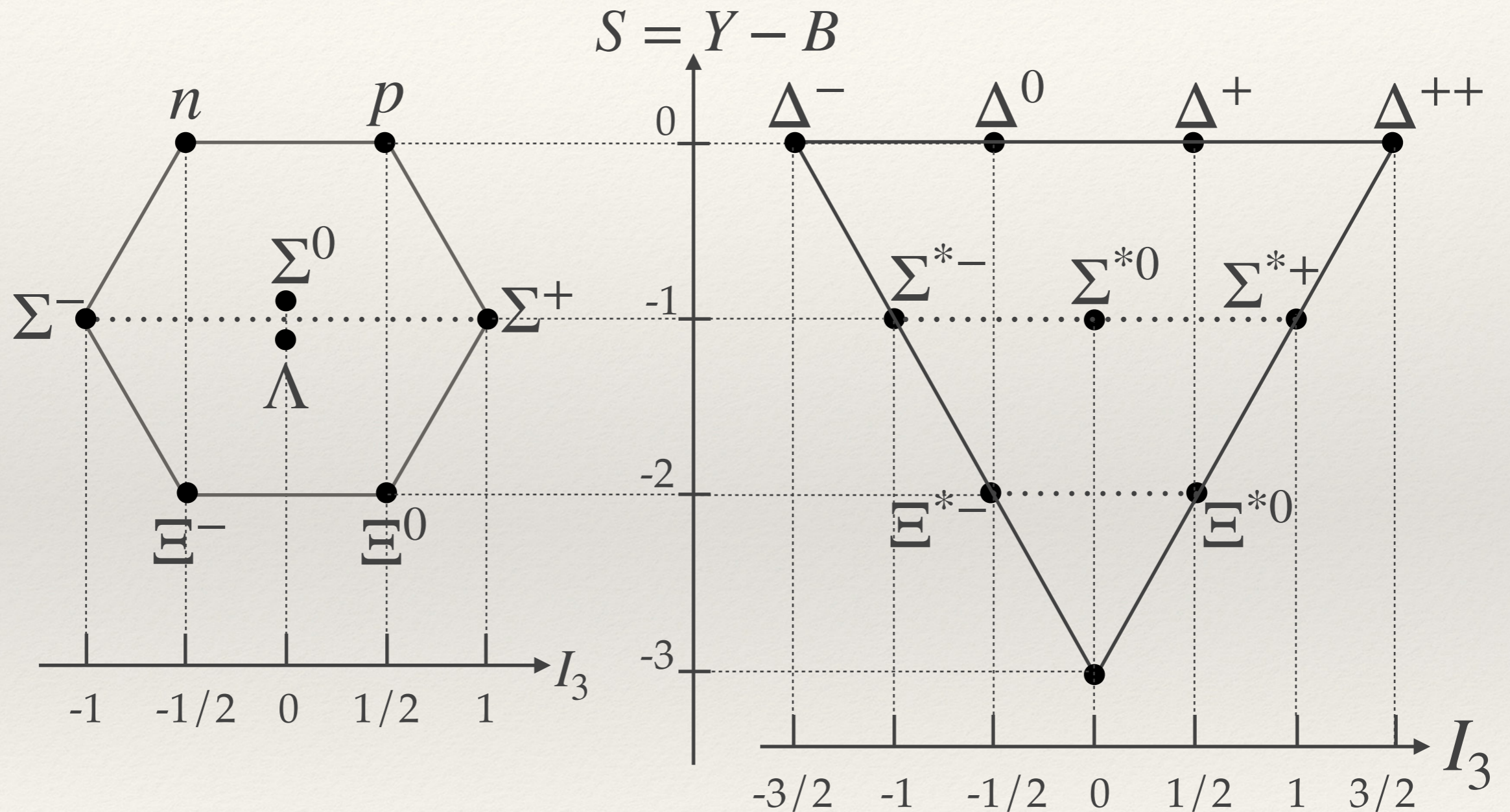
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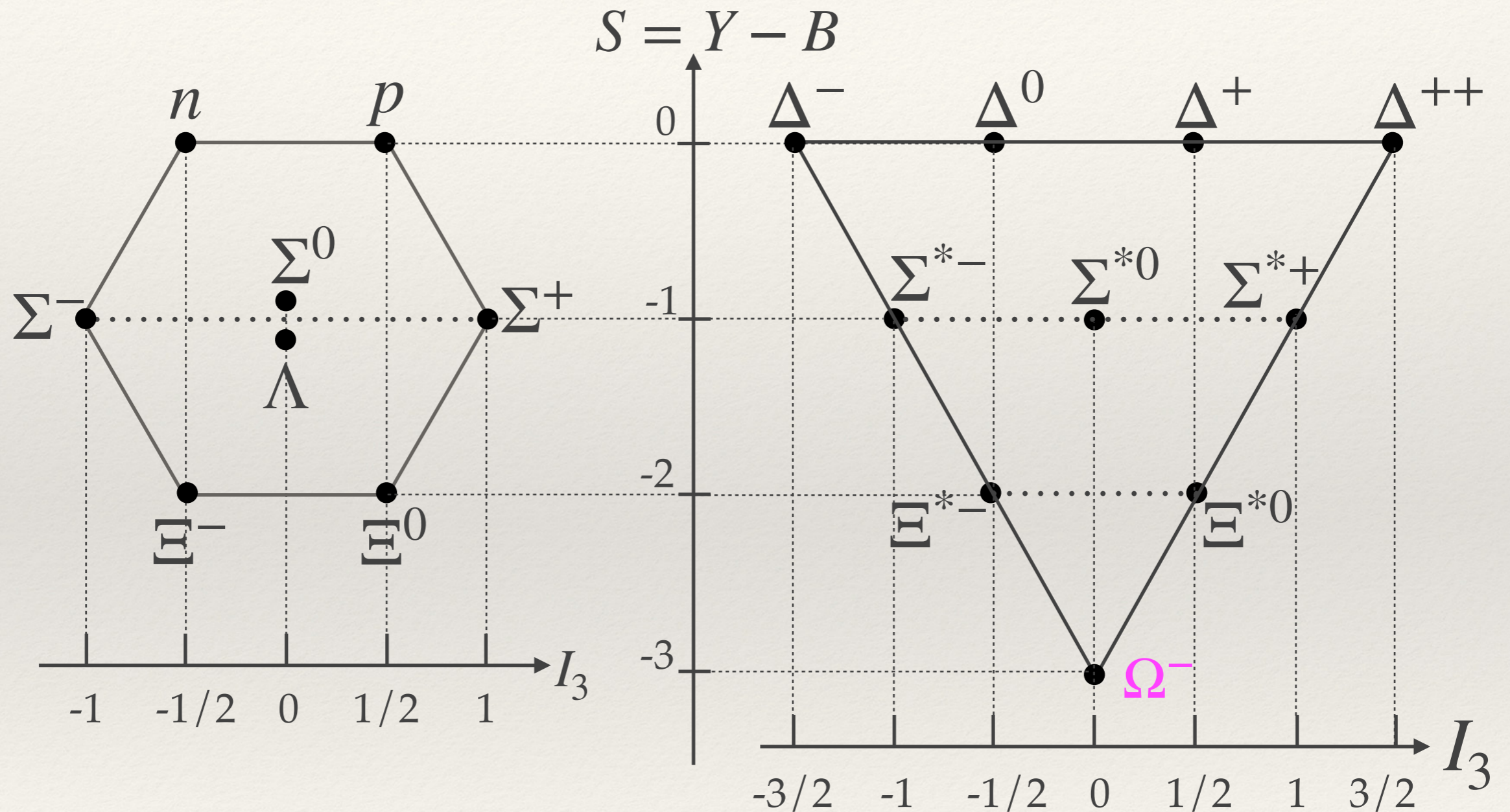
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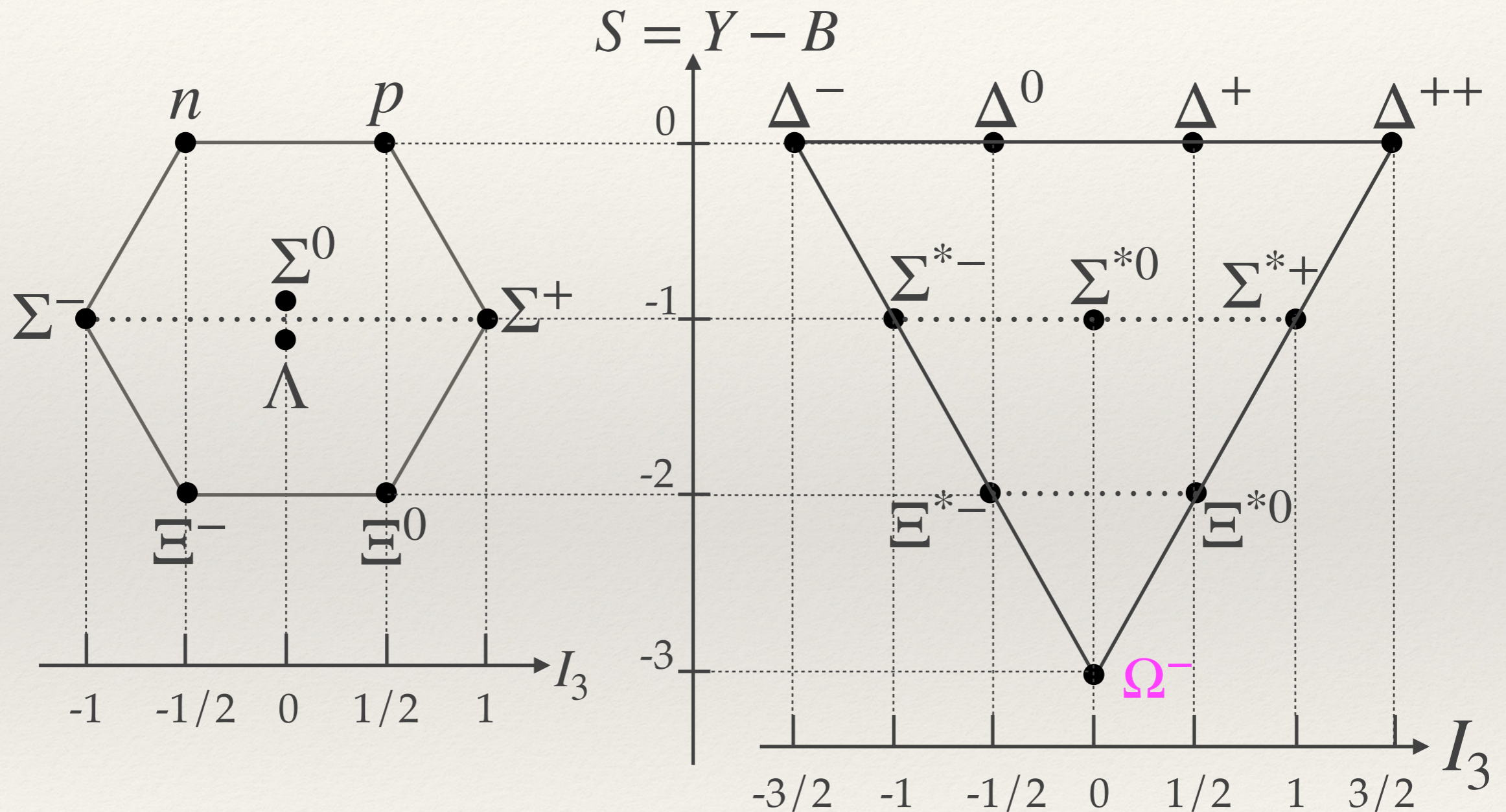
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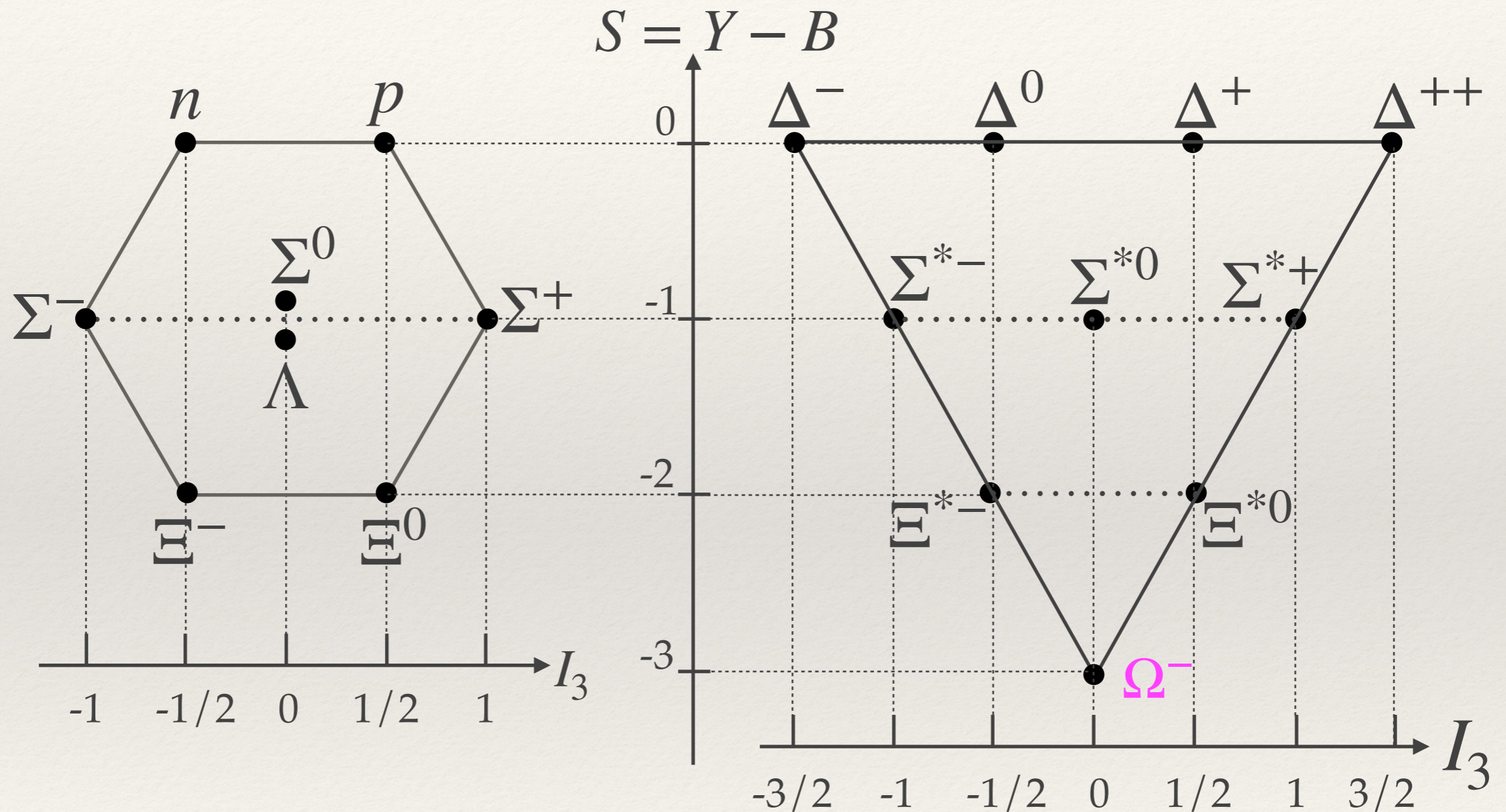


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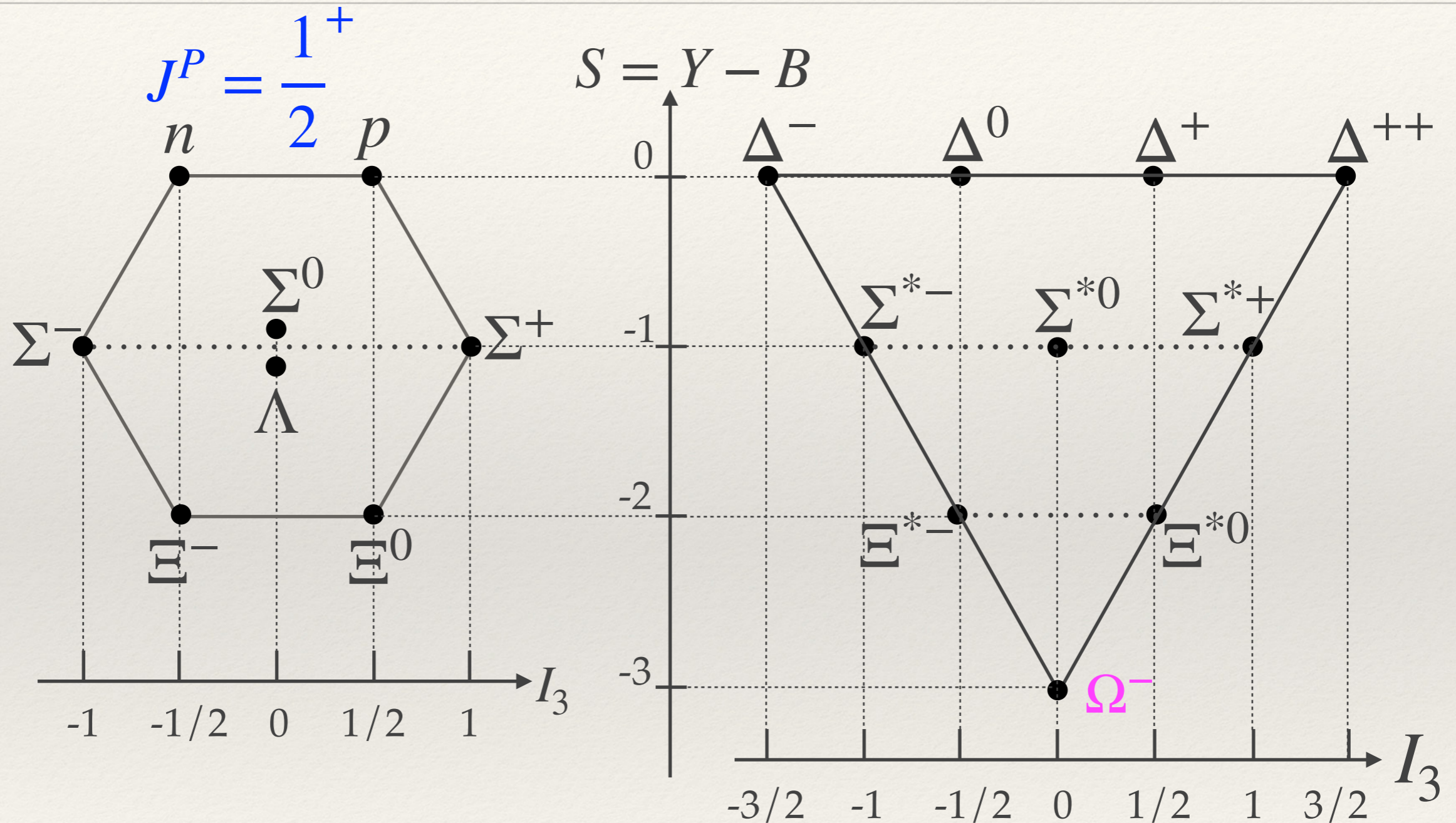
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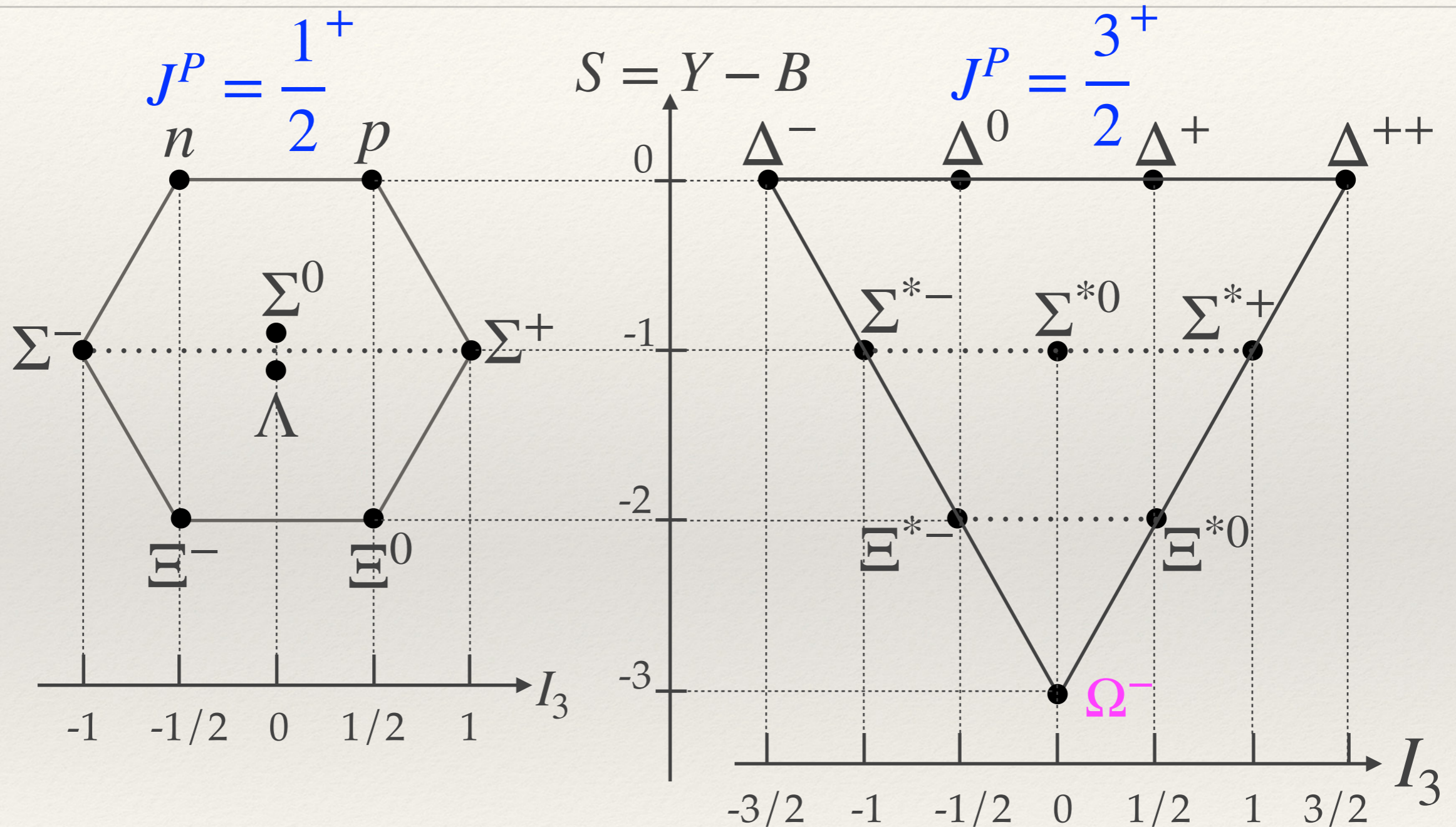
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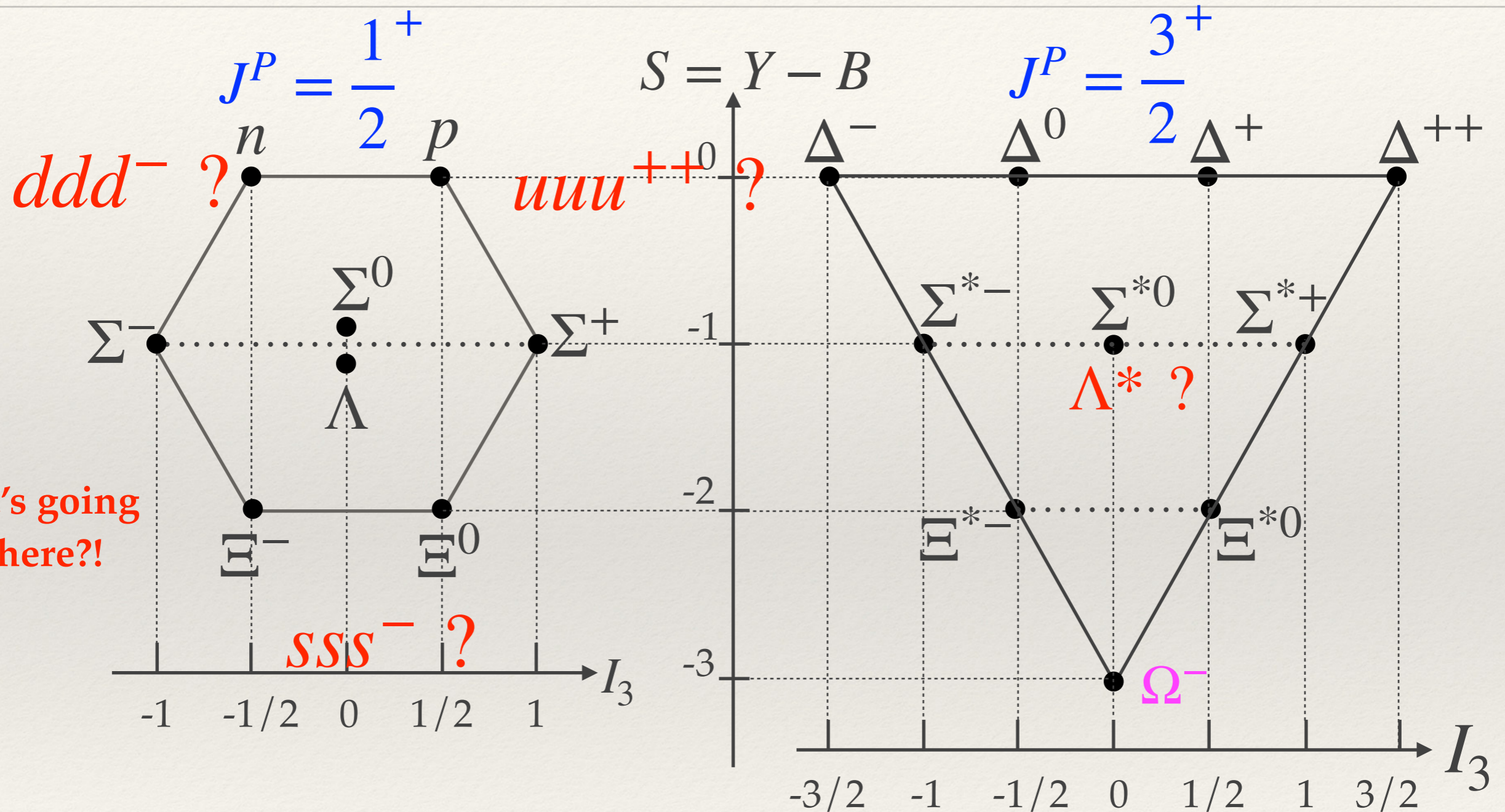
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# Baryon quark-spin wave functions

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- ❖ One elegant way to explain this pattern is to assume that the wavefunction for identical quarks (fermions) is symmetric - **exactly the opposite of what we expect for identical fermions (The Pauli Principle)!**
- ❖ Since  $L_{12}=L_3=0$  makes the space part of the wavefunction symmetric, the spin part must also be symmetric (**under this curious assumption**).

$$\Psi = \psi_{space}(\vec{r})\chi_{spin}$$

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# Spin $1/2+1/2$ wavefunctions

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S=1 symmetric  
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All pairs of identical quarks in the baryon must be in S=1!

# Baryon quark-spin wavefunctions

- ❖ For  $\Delta^{++}(uuu)$ ,  $\Delta^{-}(ddd)$ ,  $\Omega^{-}(sss)$  the only way to make all pairs spin-symmetric is with all 3 spins parallel, i.e.  $J=3/2$ , i.e. no  $J=1/2$  states.

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$$S = 1 + \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$$

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- ❖ which gives us:  $n, p$  ( $J = \frac{1}{2}$ ) and  $\Delta^0, \Delta^+$  ( $J = \frac{3}{2}$ )

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$$\Sigma^-, \Sigma^+ \left(J = \frac{1}{2}\right) \text{ and } \Sigma^{*-}, \Sigma^{*+} \left(J = \frac{3}{2}\right)$$

- ❖ The  $uds$  state has the  $ud$  in  $S=1$  by isospin symmetry. Adding the  $s$  quark gives us

$$\Sigma^0 \left(J = \frac{1}{2}\right) \text{ and } \Sigma^{*0} \left(J = \frac{3}{2}\right)$$

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# Baryon quark-spin wavefunctions

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- ❖ An orthogonal  $uds$  state has the  $ud$  in  $S=0$ . Adding the  $s$  quark:

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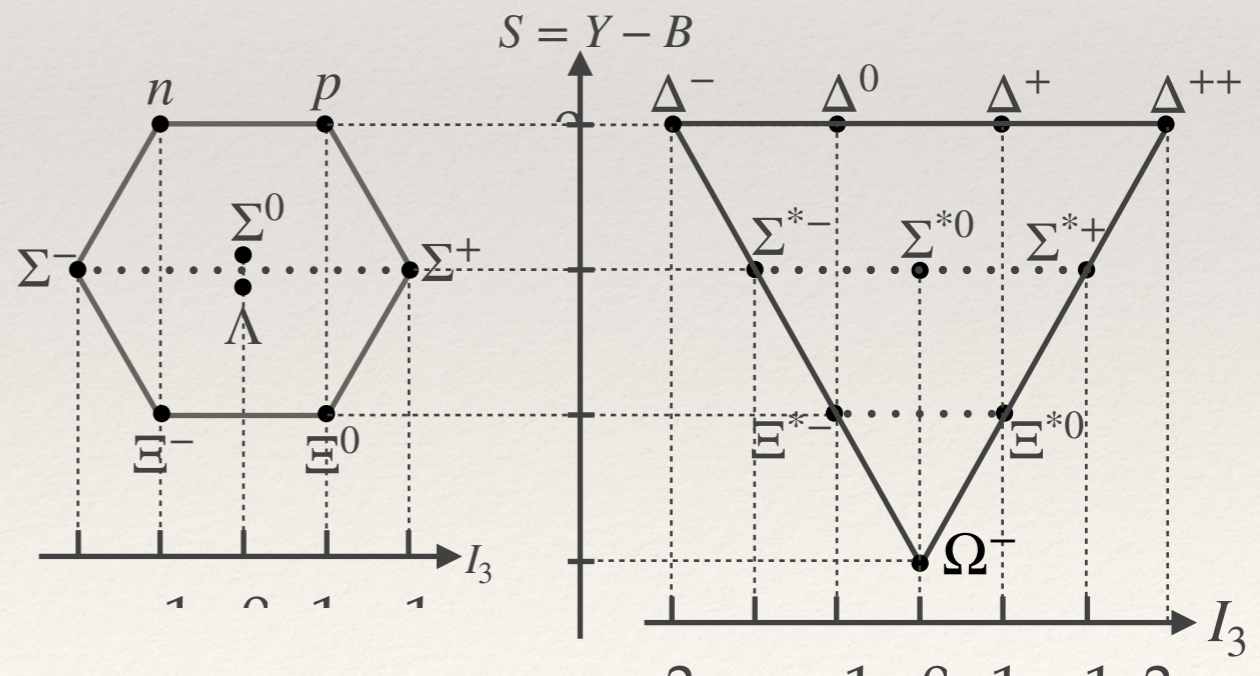
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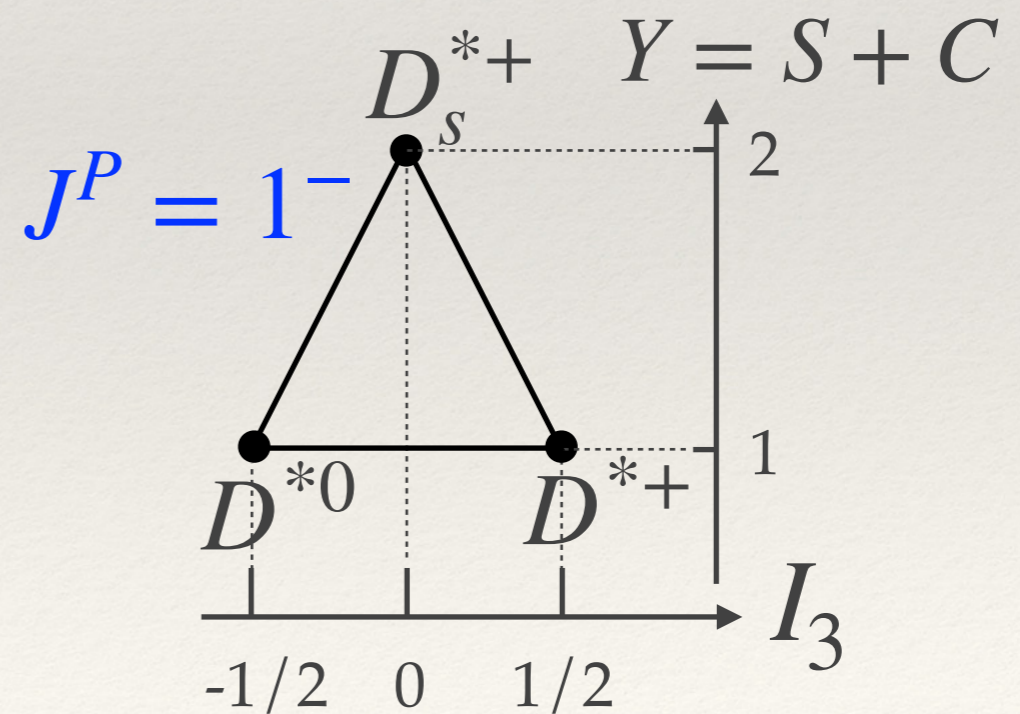
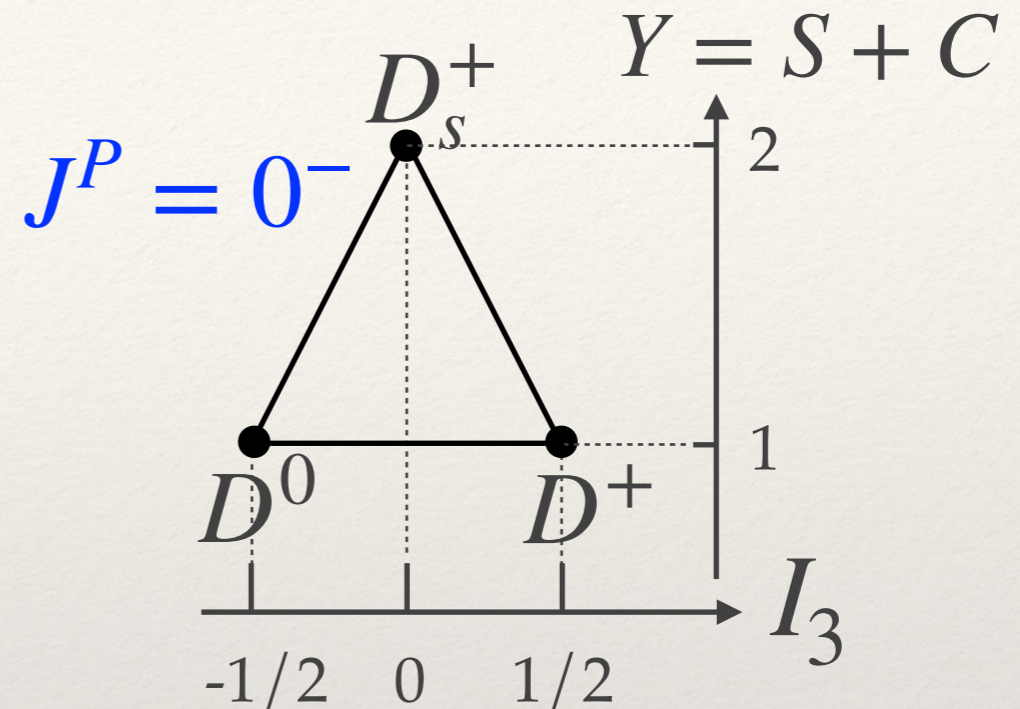
$$\Lambda(J = \frac{1}{2})$$

- ❖ We have now explained the pattern



# Heavy quark states ( $b, c$ )

- ❖ Similar patterns of allowed states for  $J=0,1$  mesons,  $J=1/2, 3/2$  baryons with 1,2,3 heavy quarks. Not yet all are seen but LHCb and other experiments are slowly but steadily finding them.
- ❖ Work out the quark content of the mesons to the right.





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$$\Psi = \psi_{space}(\vec{r})\chi_{spin}\chi_{color}$$

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P.S. Typo in M&S Table 6.13

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# Color confinement 18.03.2019

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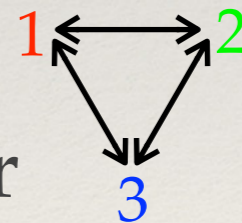
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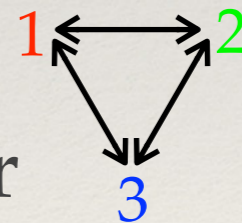
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$$\chi_B^C = \frac{1}{\sqrt{6}}(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3)$$

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# Quark combinations allowed by color

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- ❖ Color wave function for arbitrary collection of quarks and antiquarks

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- ❖ Confinement condition (no net color charge)

$$I_3^C = \frac{(\alpha - \bar{\alpha})}{2} - \frac{(\beta - \bar{\beta})}{2} \quad Y^C = \frac{(\alpha - \bar{\alpha})}{2} + \frac{(\beta - \bar{\beta})}{2} - \frac{2(\gamma - \bar{\gamma})}{3}$$

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$$I_3^C = \frac{(\alpha - \bar{\alpha})}{2} - \frac{(\beta - \bar{\beta})}{2} \quad Y^C = \frac{(\alpha - \bar{\alpha})}{2} + \frac{(\beta - \bar{\beta})}{2} - \frac{2(\gamma - \bar{\gamma})}{3}$$

- ❖ Allowed numbers of quarks and antiquarks

$$\begin{aligned} (\alpha - \bar{\alpha}) = (\beta - \bar{\beta}) = (\gamma - \bar{\gamma}) \equiv p &\implies m_q - n_{\bar{q}} = 3p \\ &\implies (3q)^p (q\bar{q})^n \quad (p, n \geq 0) \end{aligned}$$

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\* Can you find the mistake on M&S p. 181?

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- ❖ Growing evidence for exotic hadrons with heavy quarks



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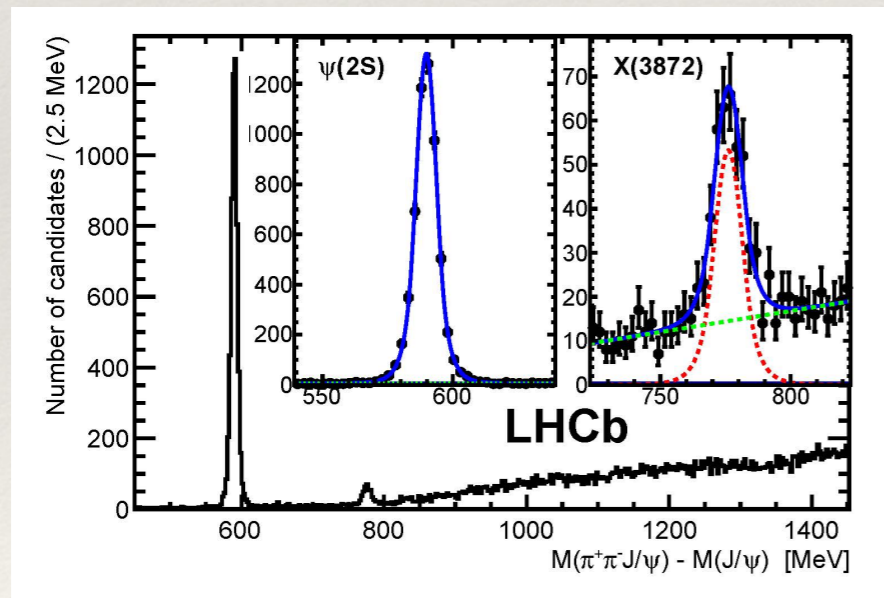
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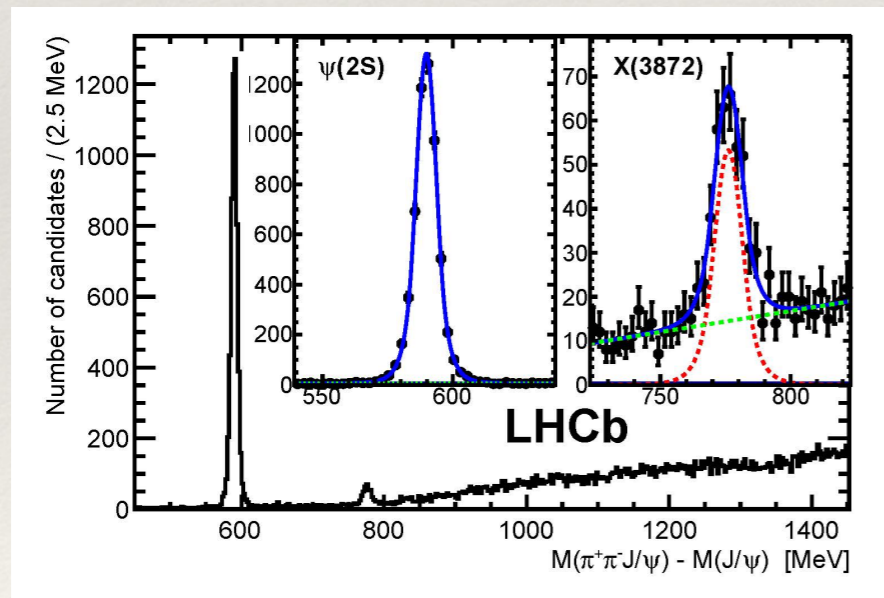


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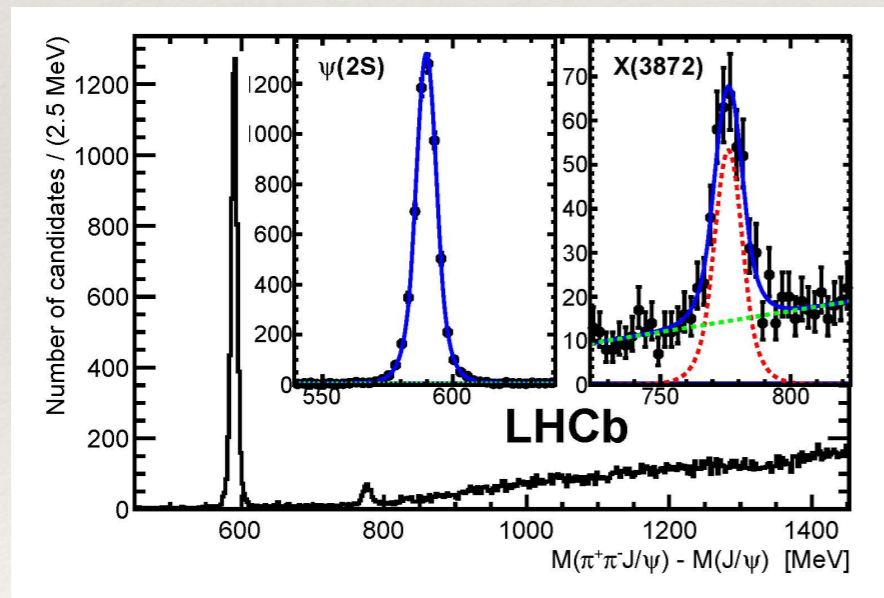
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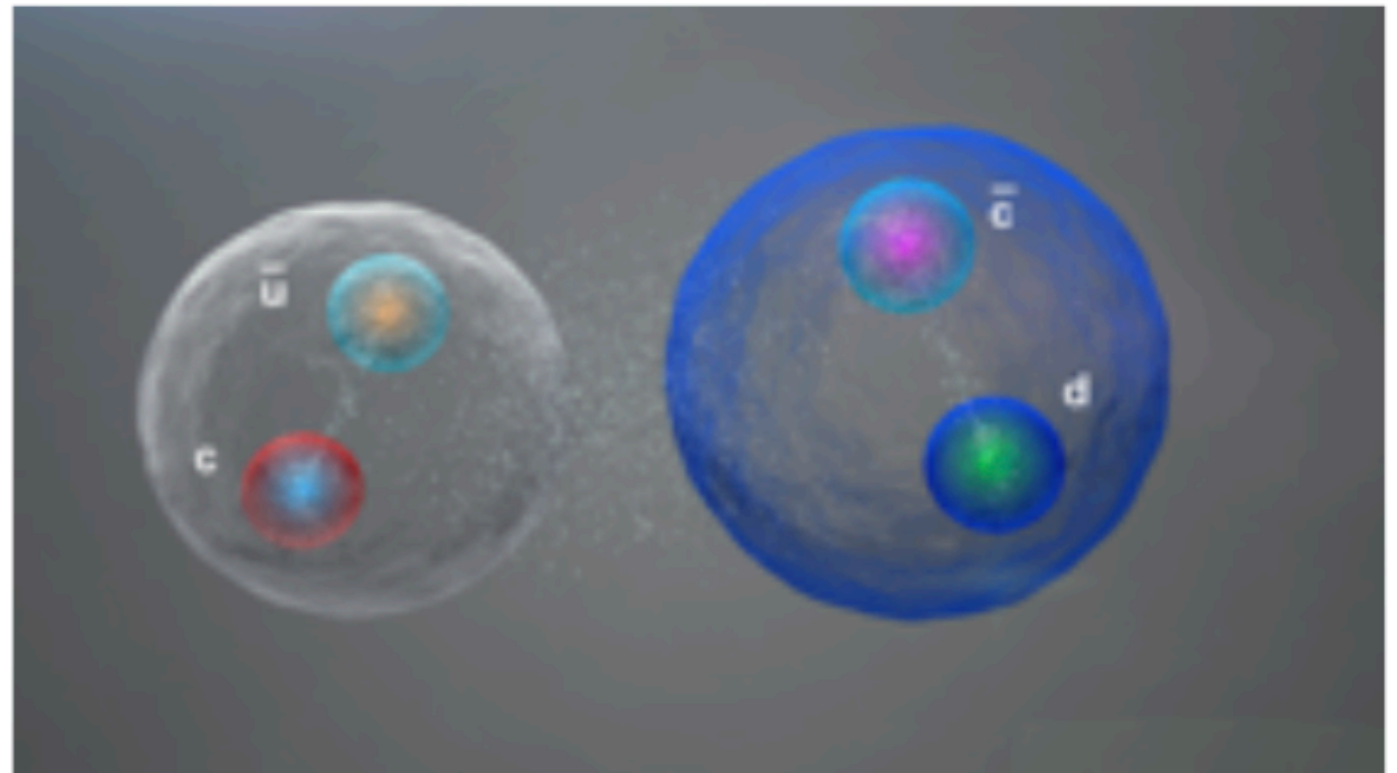
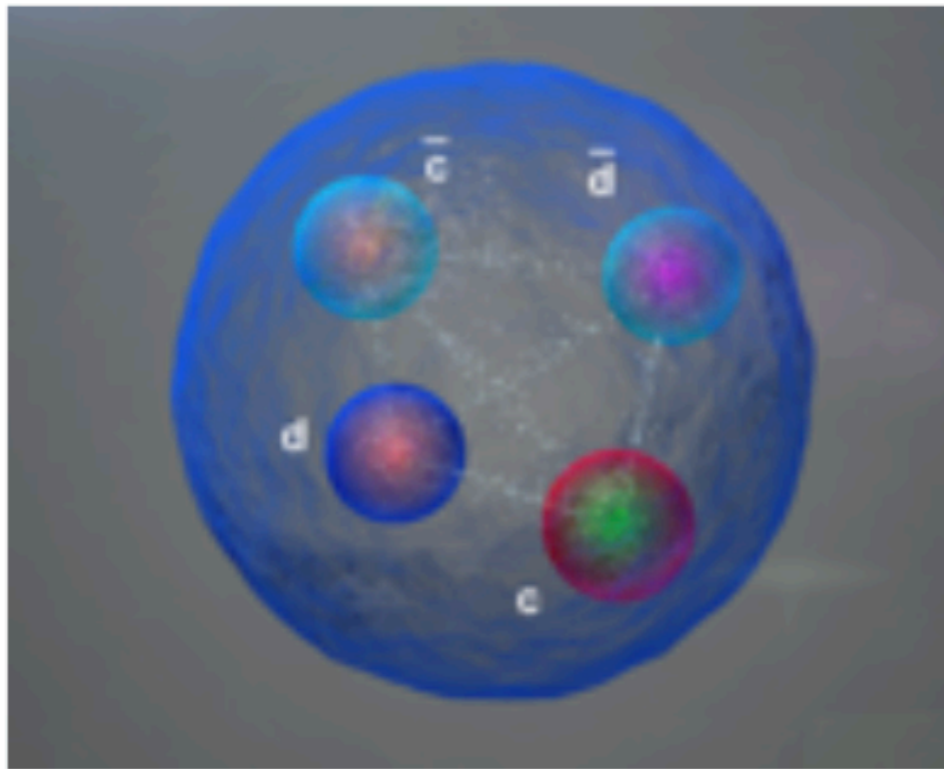
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Uses invariant mass!

Could be tetraquark or  $D\bar{D}^*$

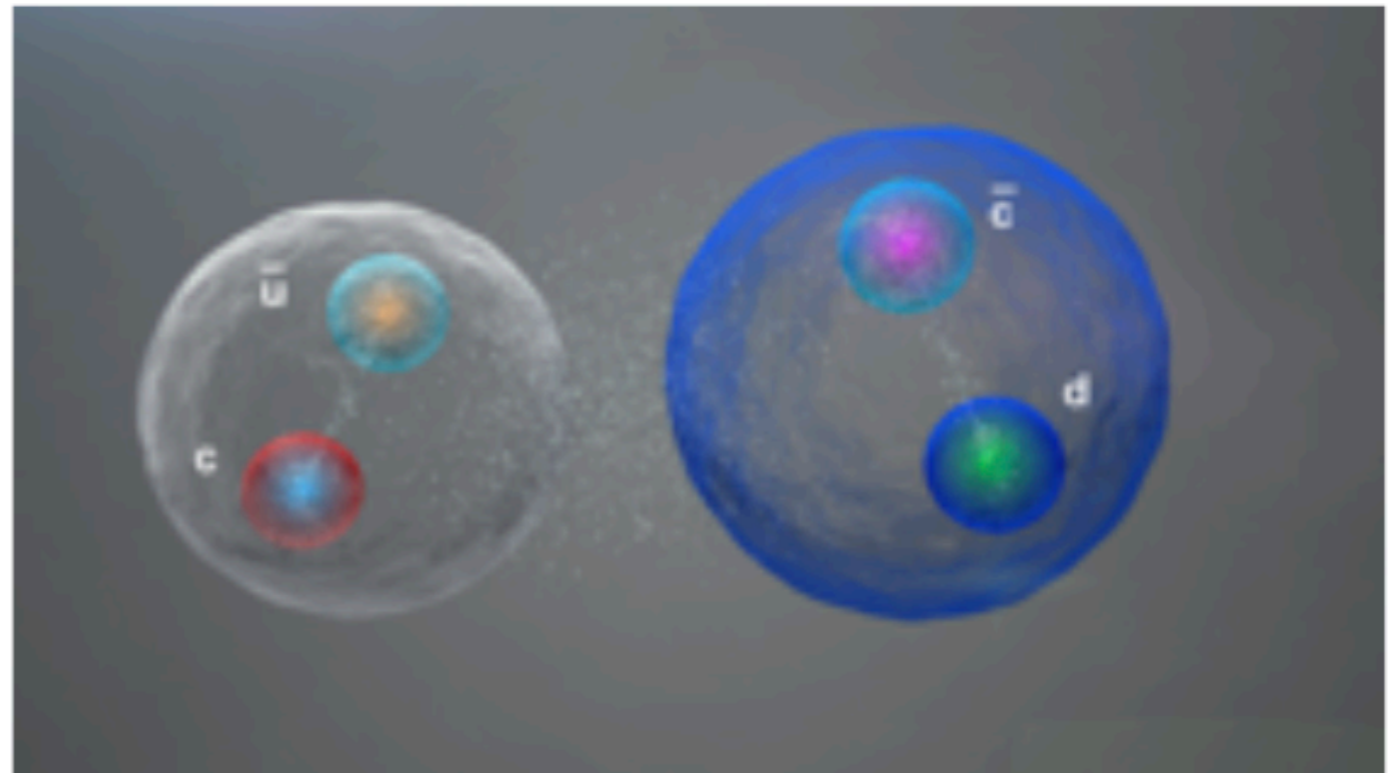
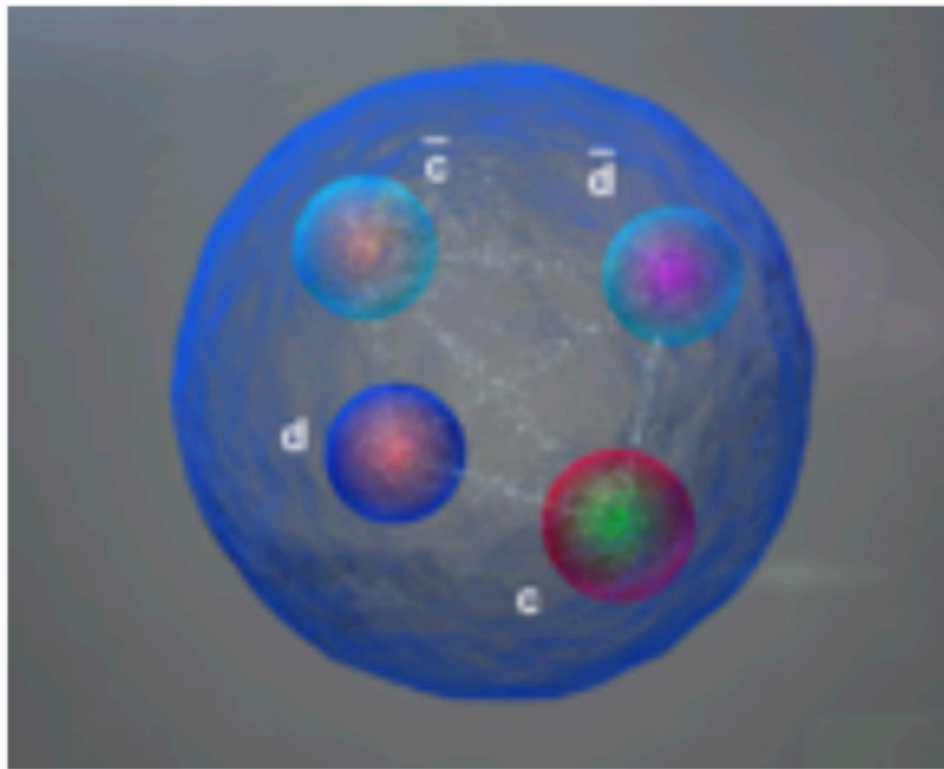
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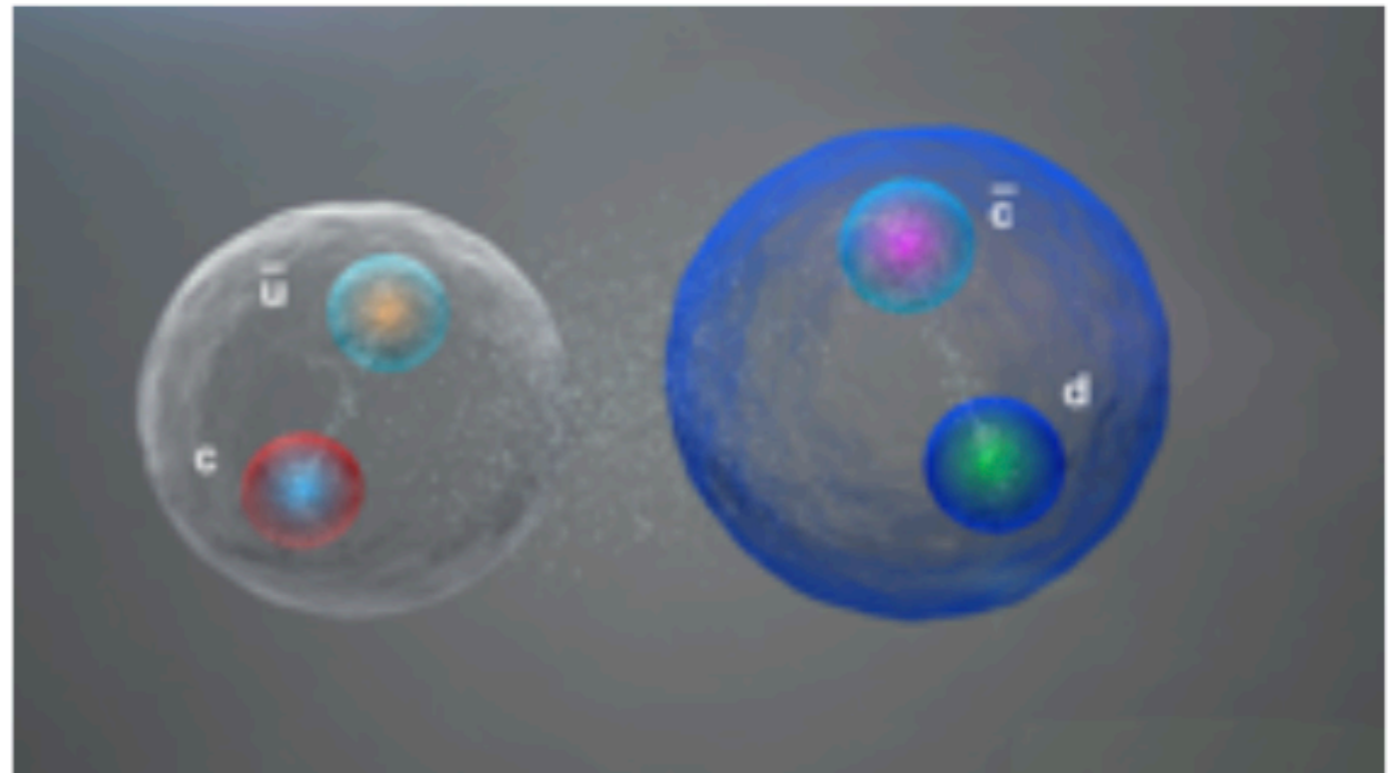
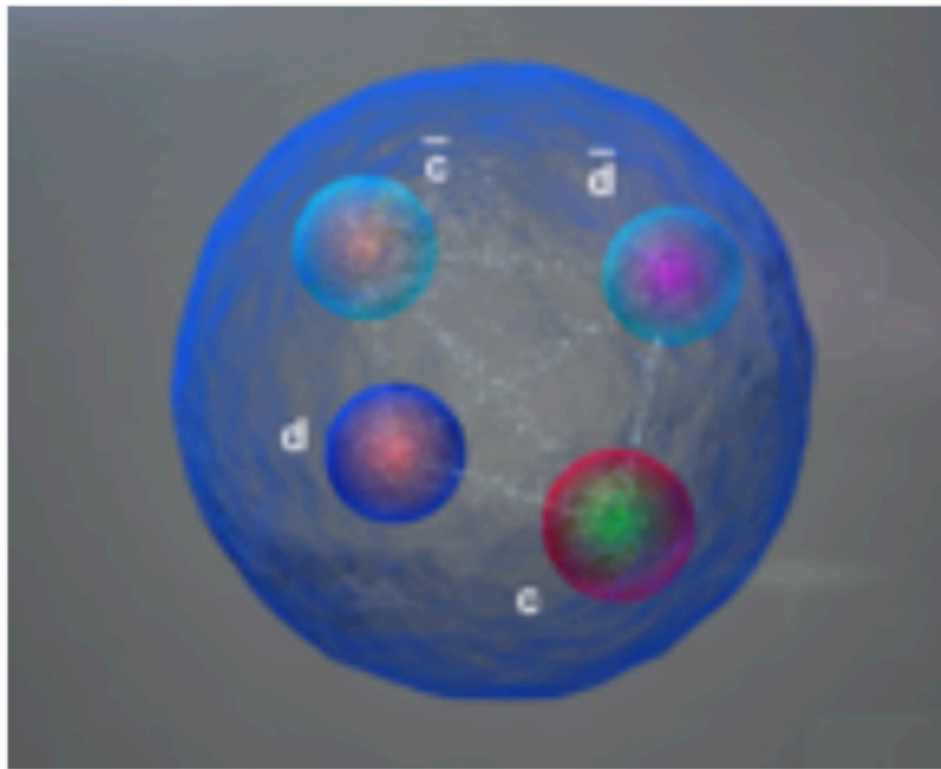
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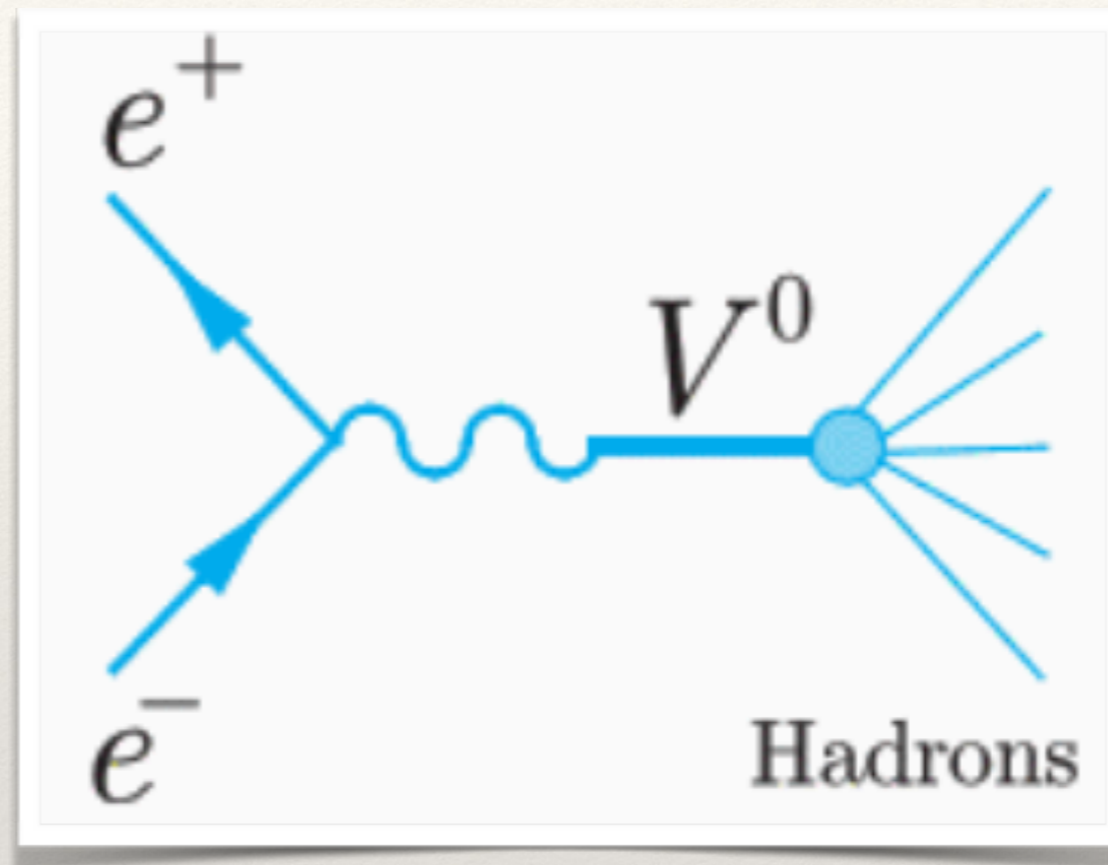
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# Charmonium and bottomonium



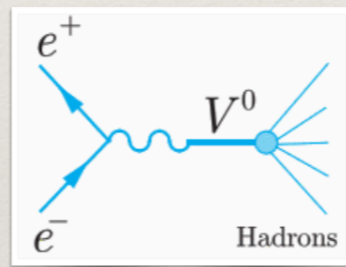
- ❖ Approximately non-relativistic bound states  $c\bar{c}$ ,  $b\bar{b}$



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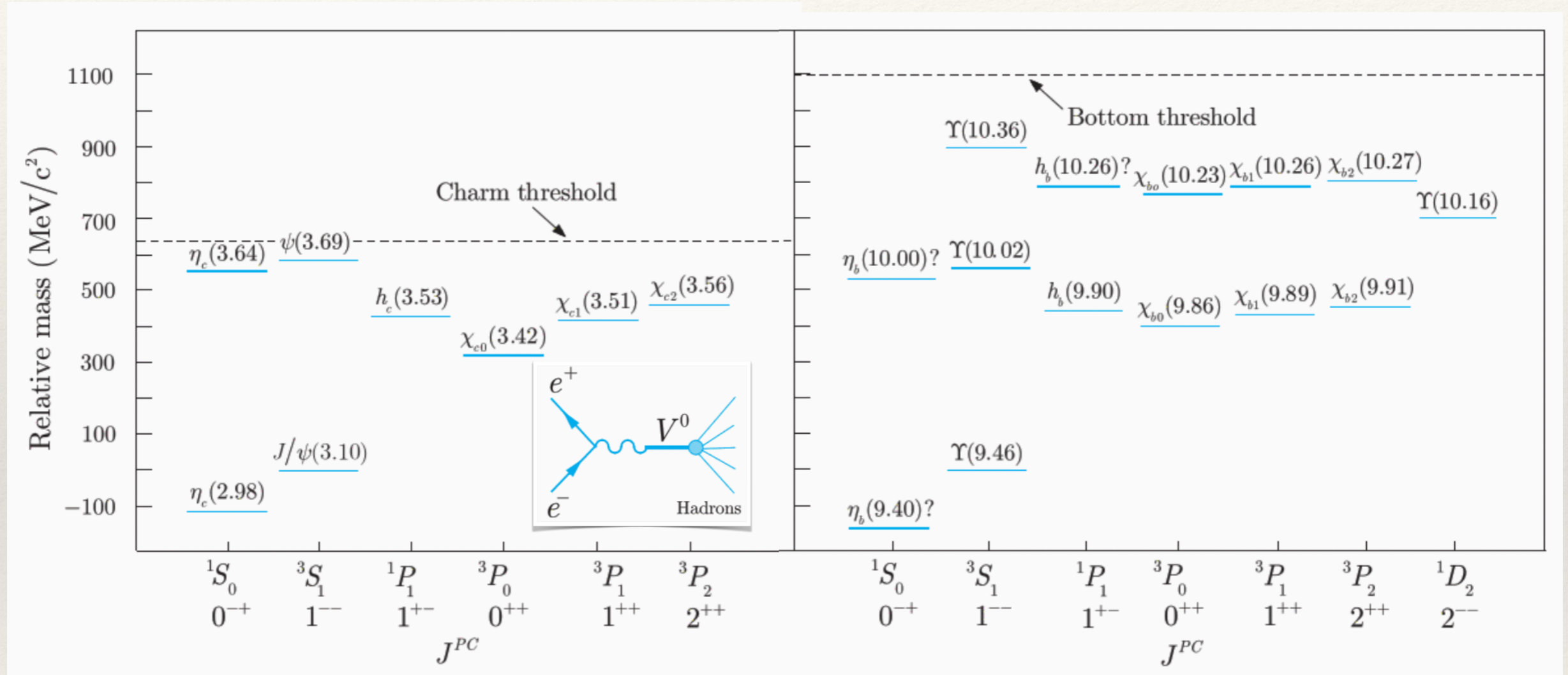
# Charmonium and bottomonium

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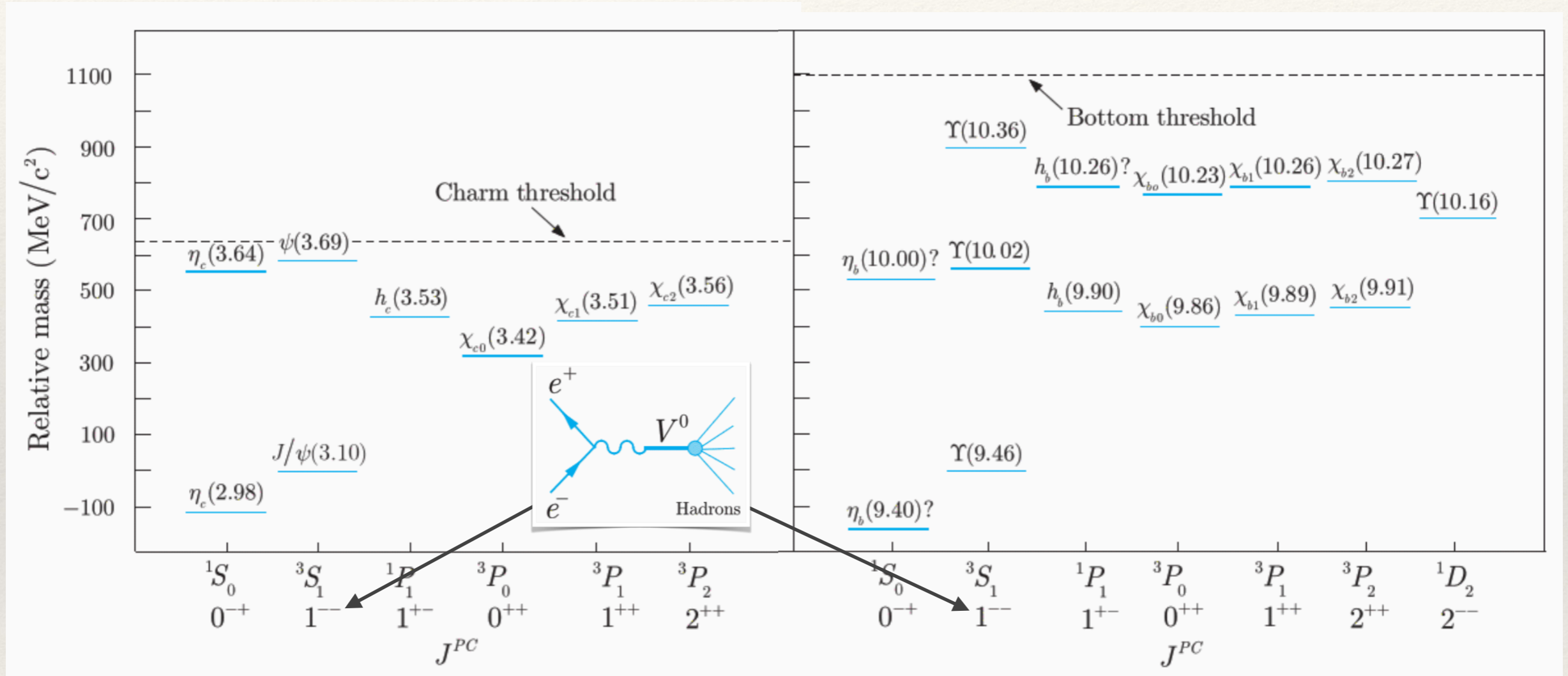
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Martin and Shaw, Particle Physics, 4th ed., Figures 6.7,9,10

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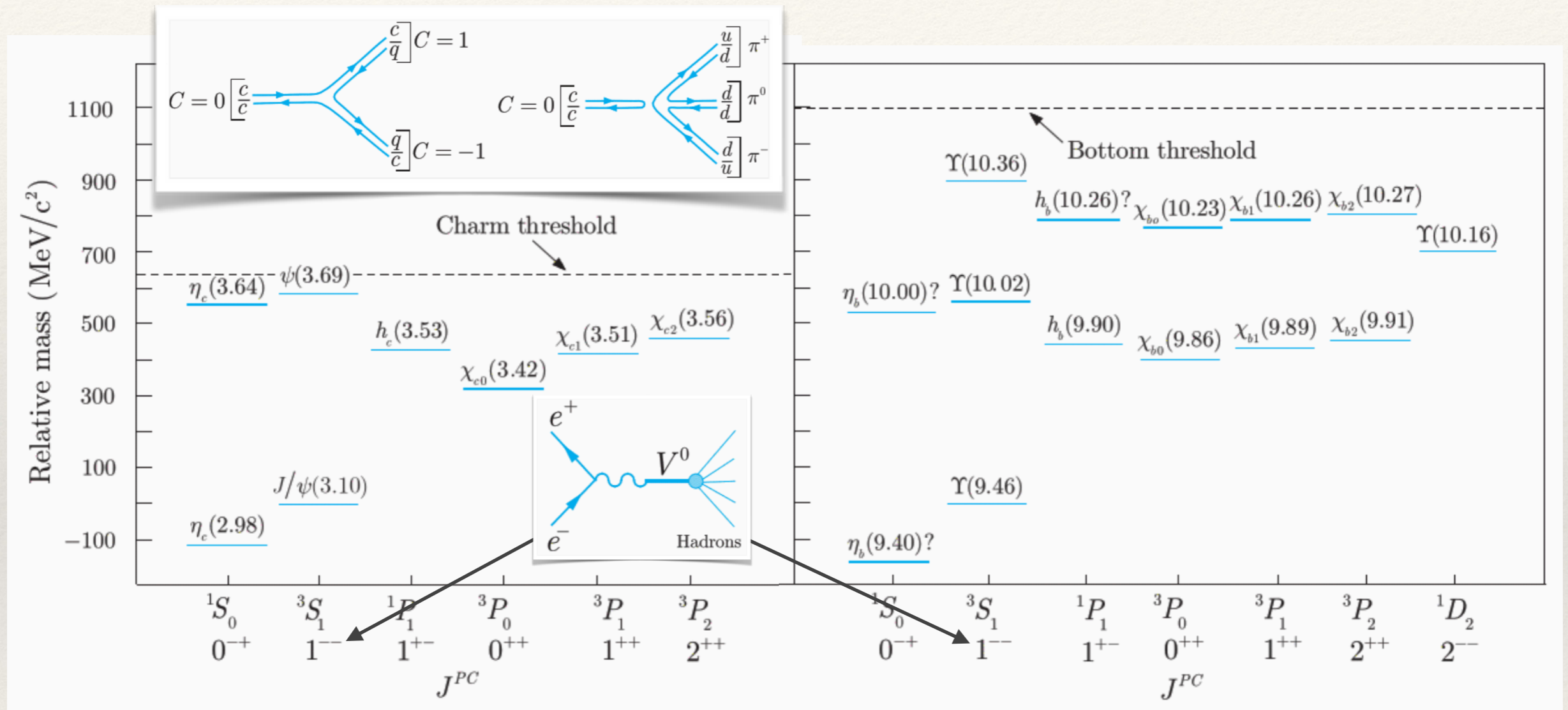
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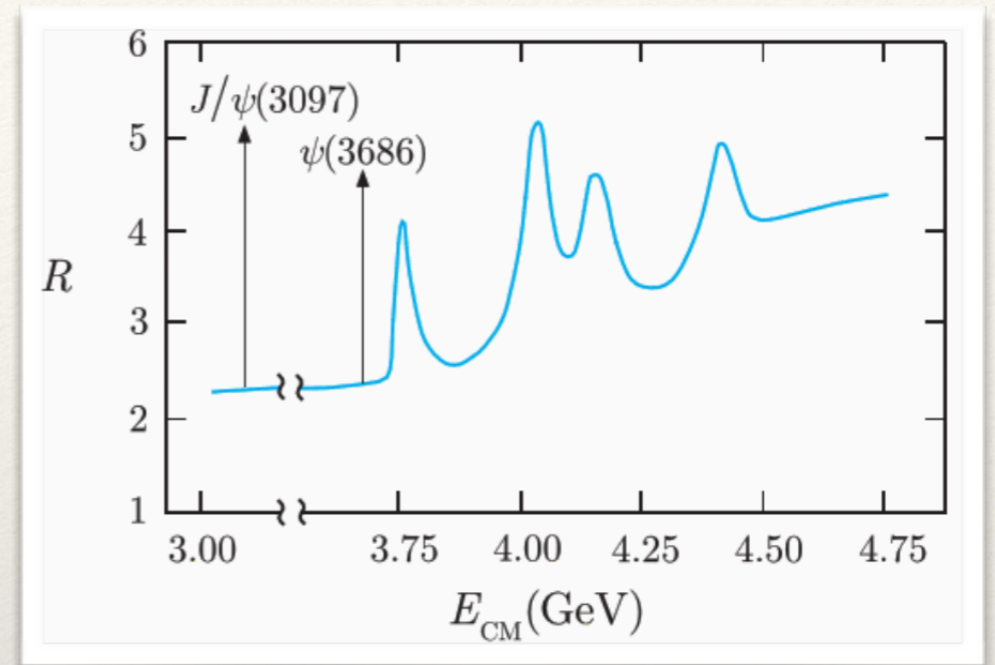
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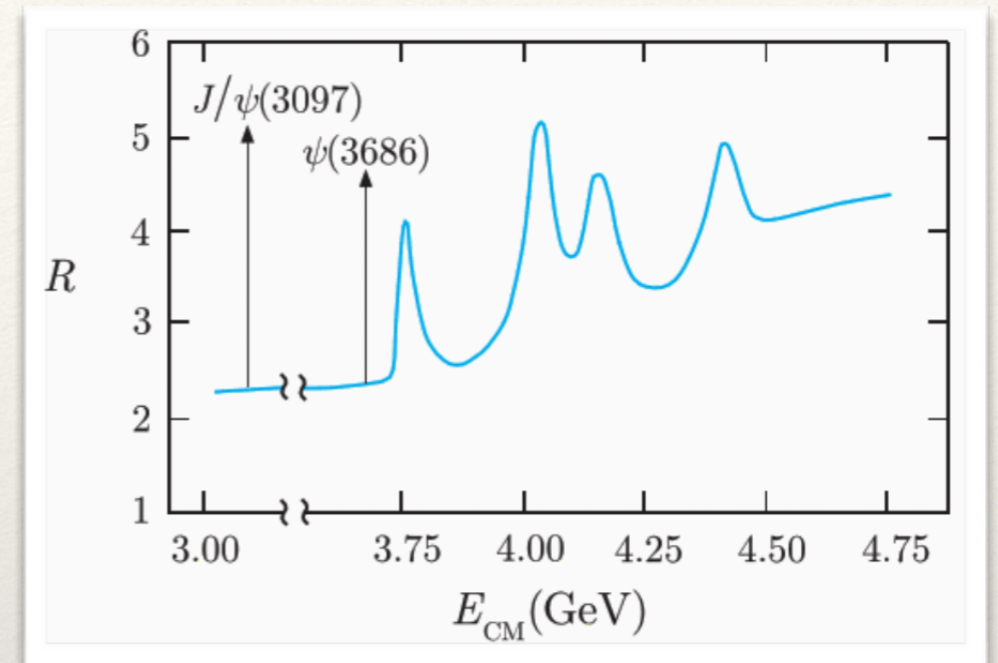
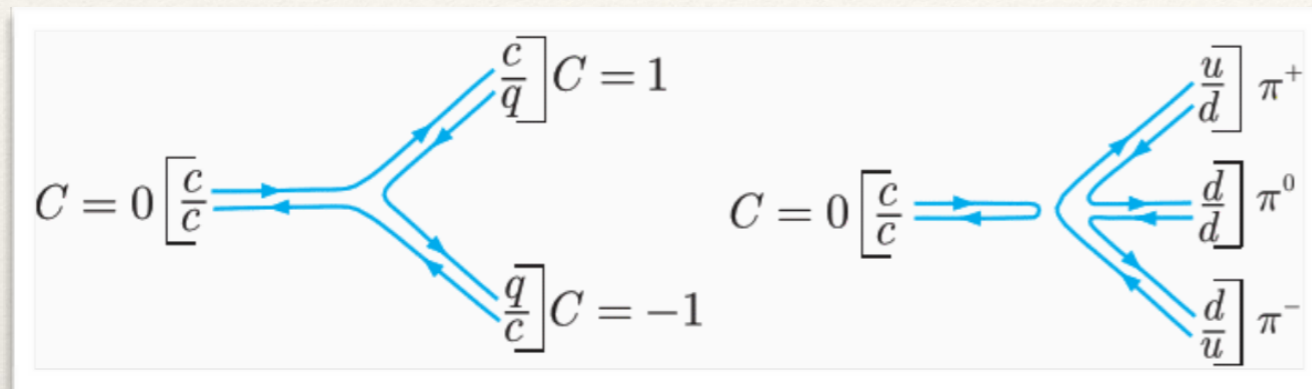
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- ❖ Suppressed annihilation of heavy quarks, long lifetime, narrow states
- ❖ Strong processes for production of open charm, short lifetime, wide states
- ❖ Detailed analysis shows that the potentials for charmonium and bottomonium are equal (confirming flavor-independent strong force)!

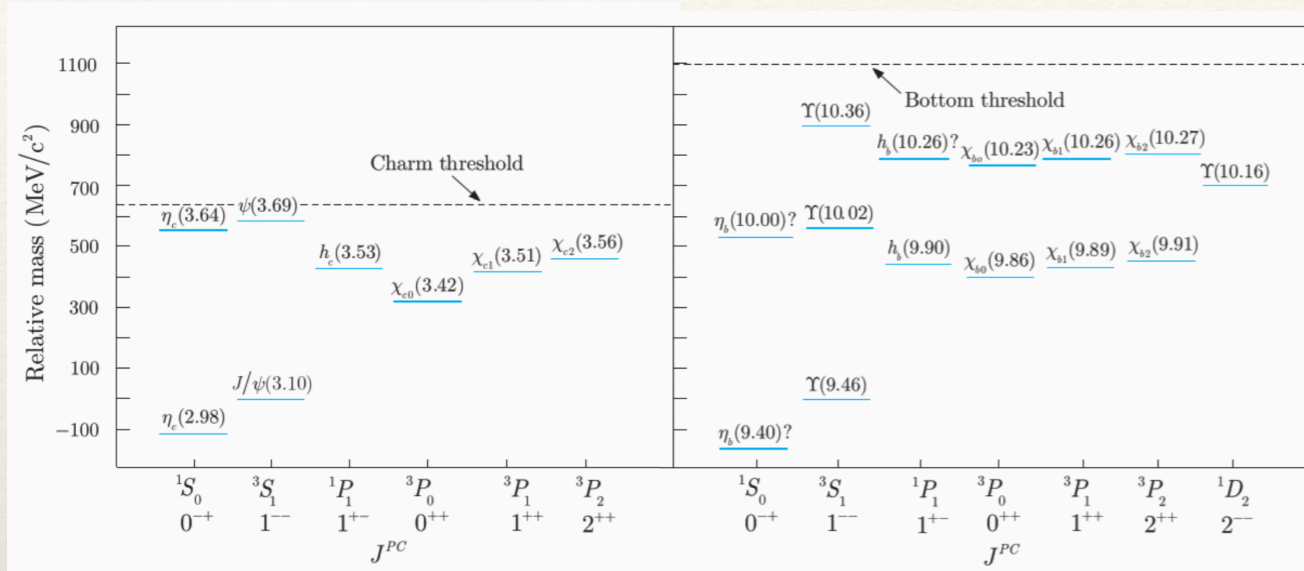
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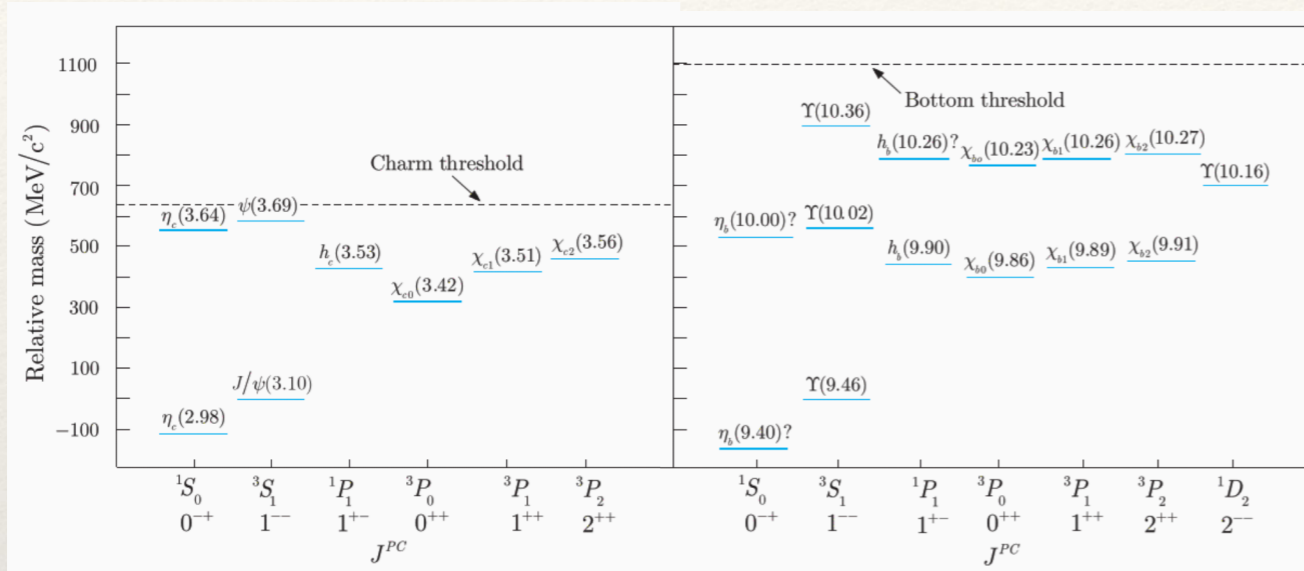
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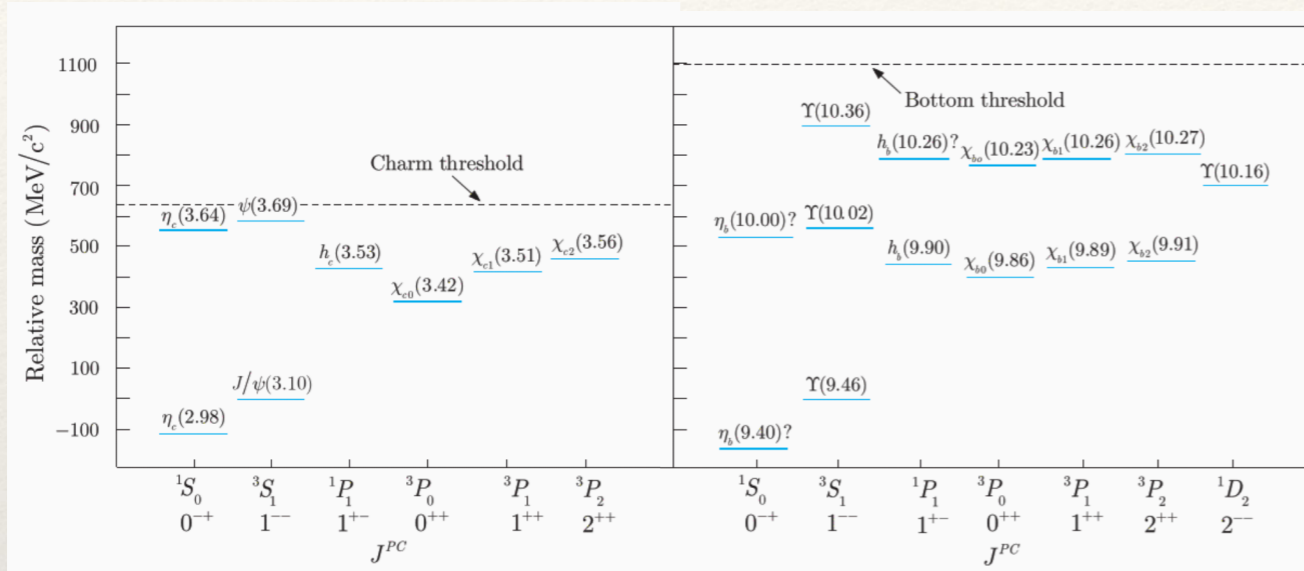


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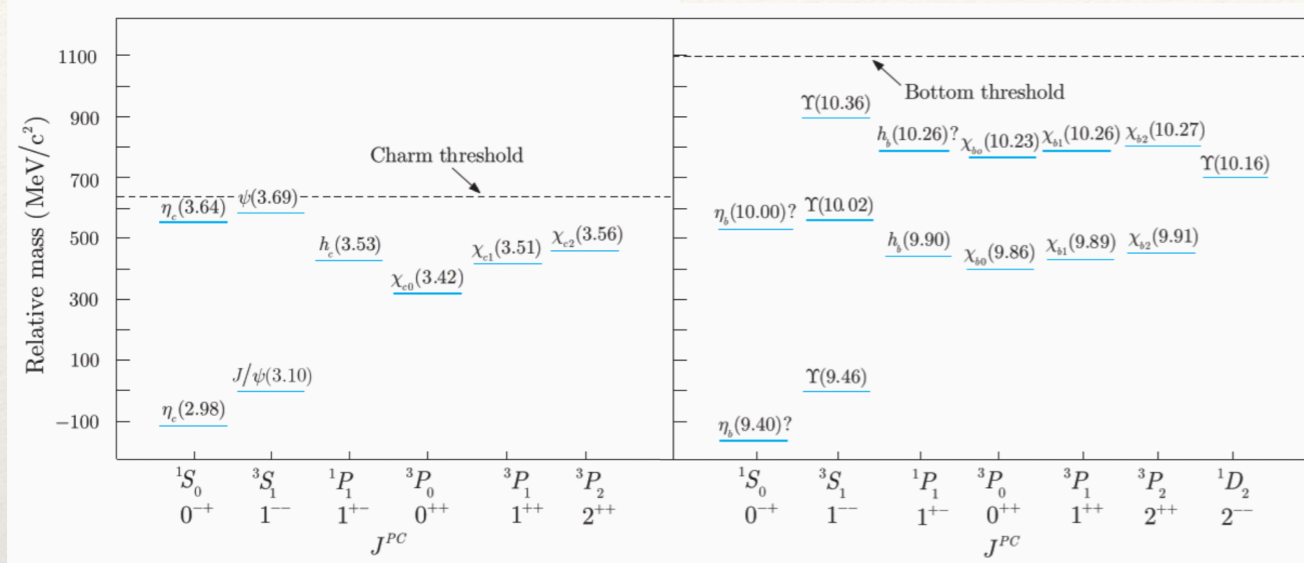
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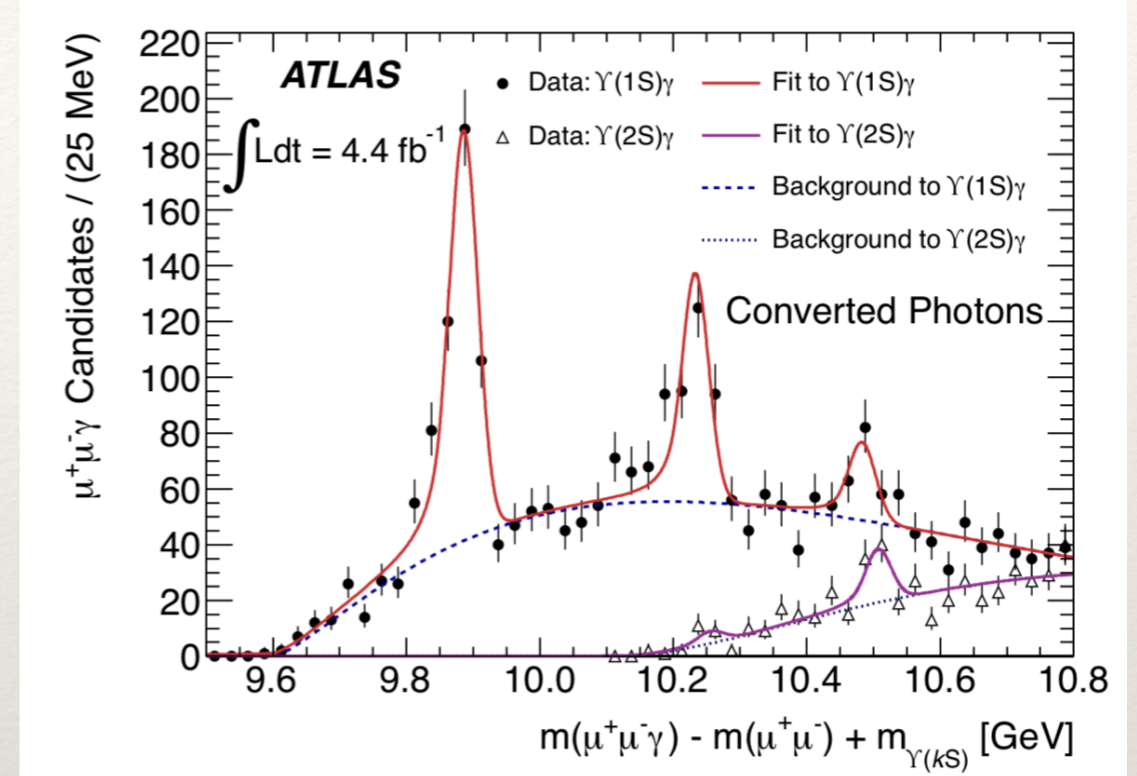
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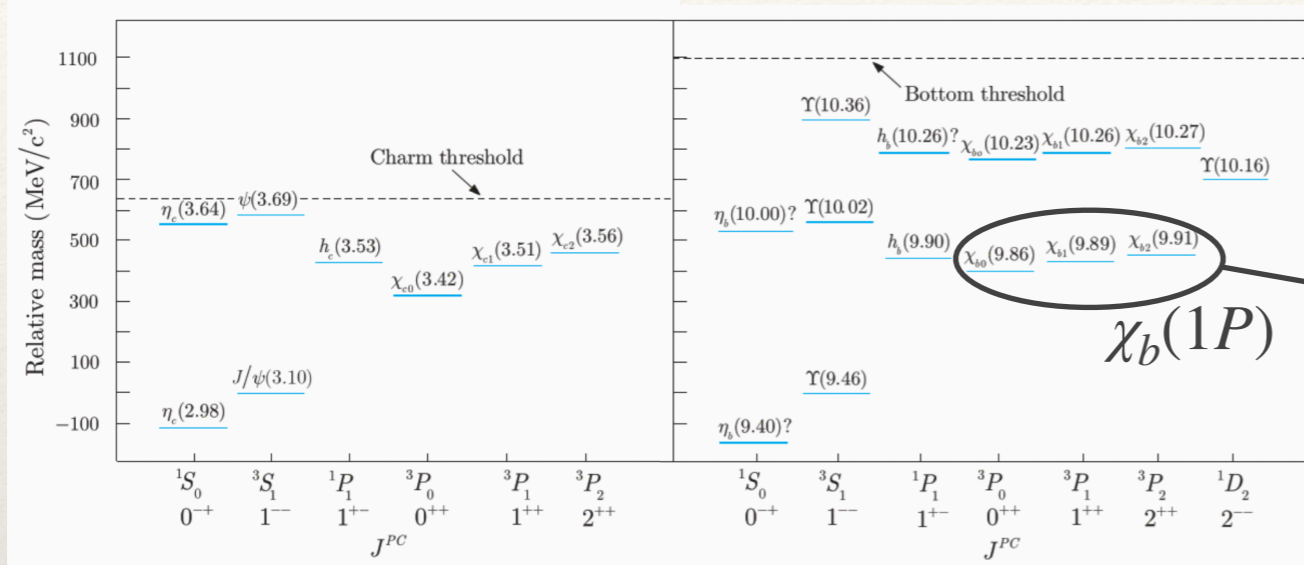
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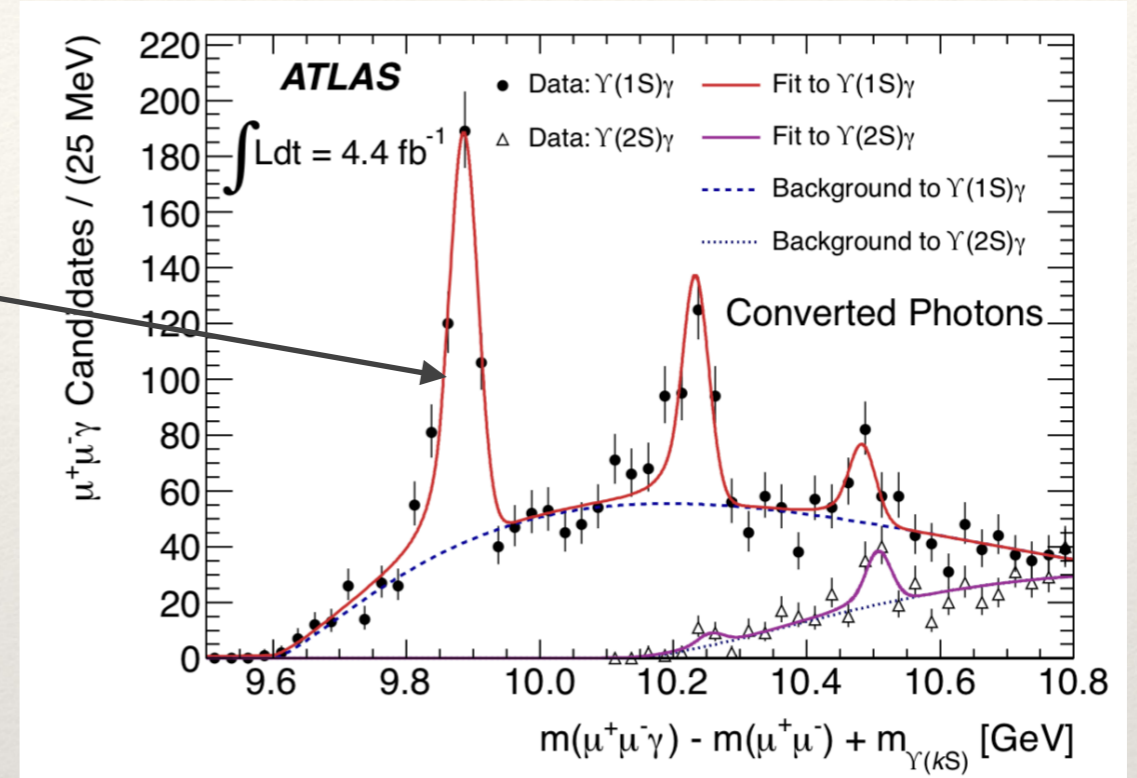
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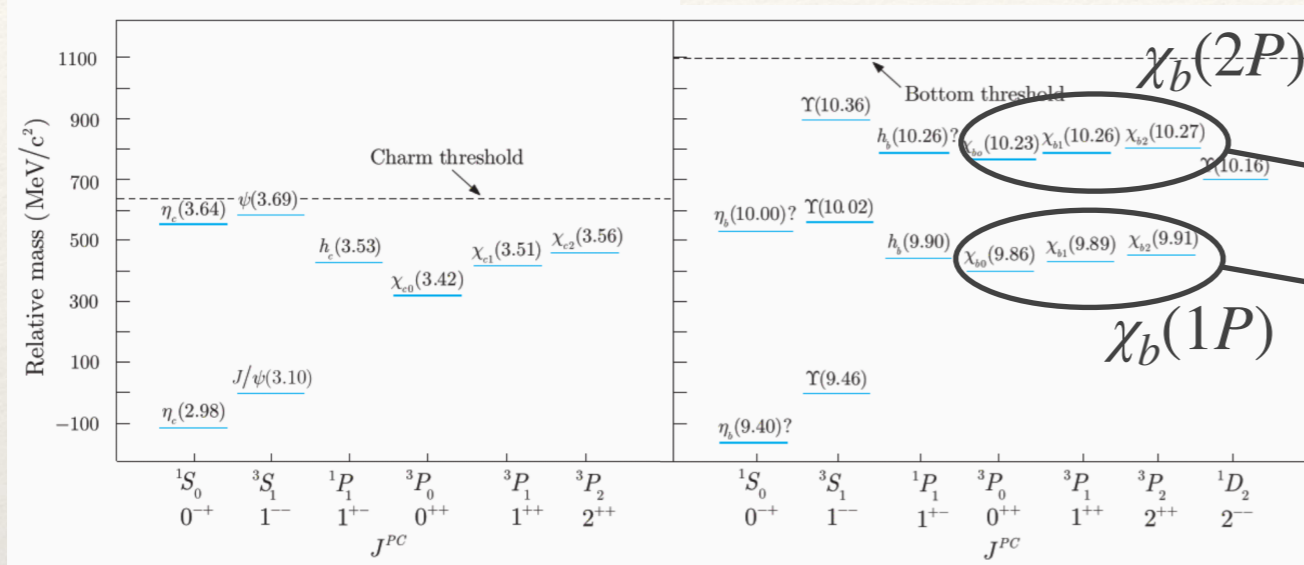


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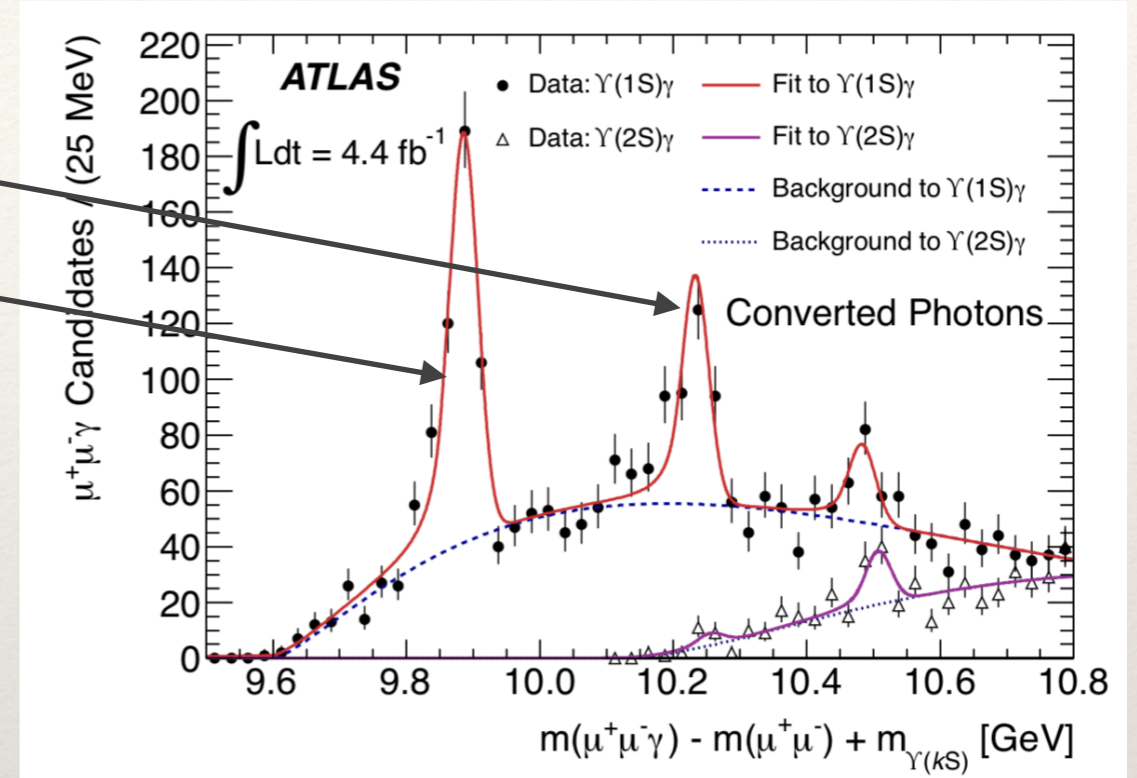
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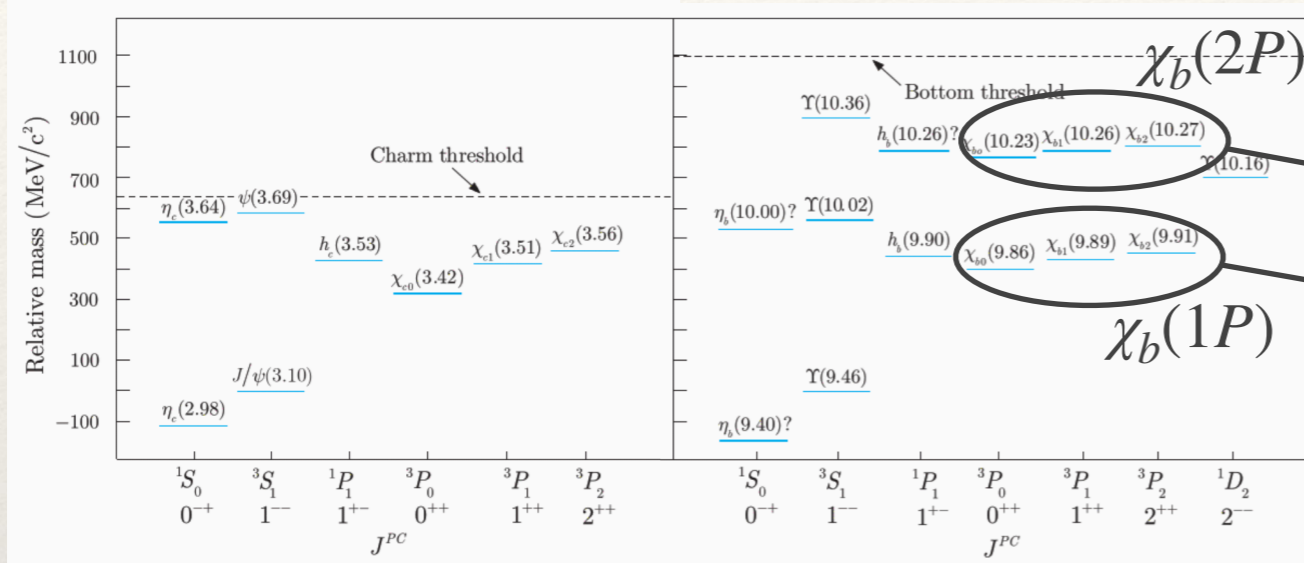


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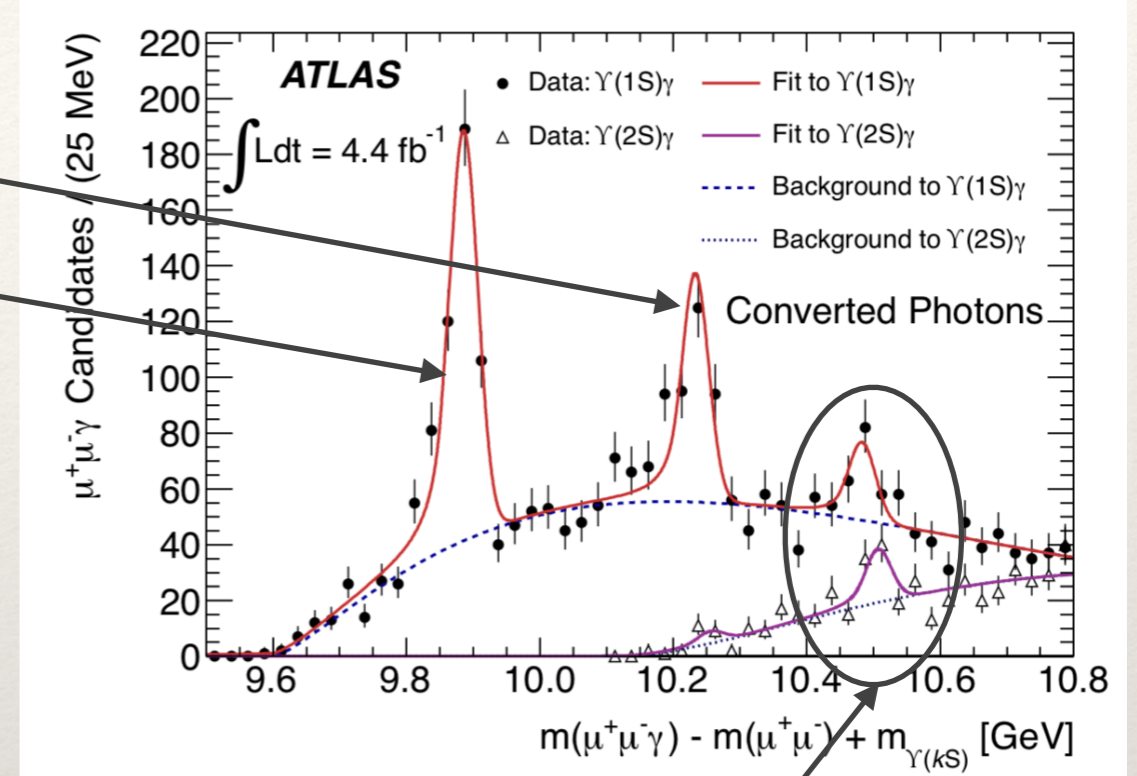
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Evidence for  $\chi_b(3P)$ !