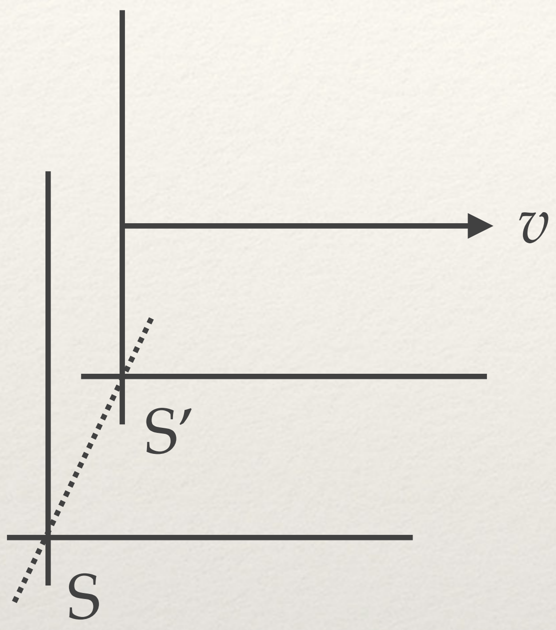
FYS3500 - spring 2019

Relativistic Kinematics*

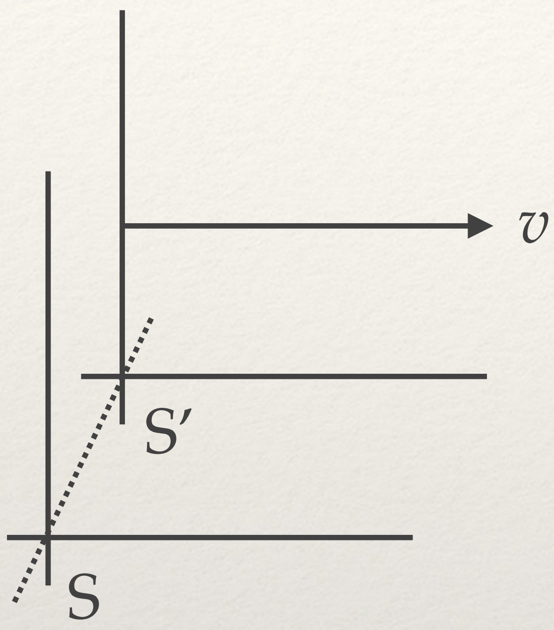
Alex Read
University of Oslo
Department of Physics

*Martin&Shaw, Particle Physics, 4th Ed., Appendix A.1,A.2 (Last update 18.02.2019 16:40)

Lorentz Transformations



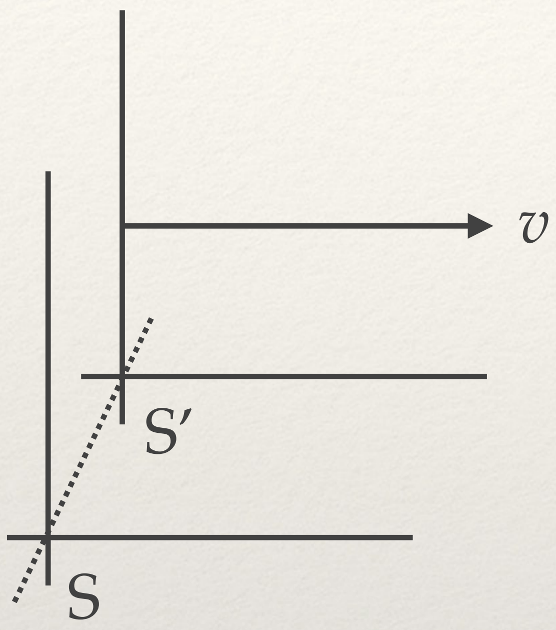
Lorentz Transformations



$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Lorentz Transformations



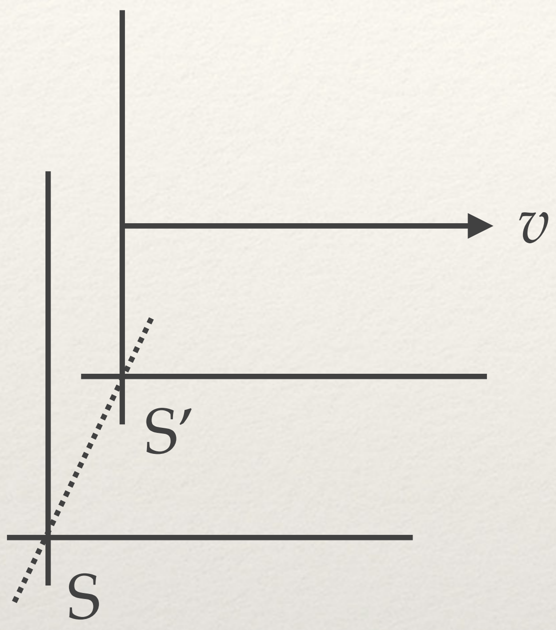
$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

Lorentz Transformations



$$\beta = \frac{v}{c}$$

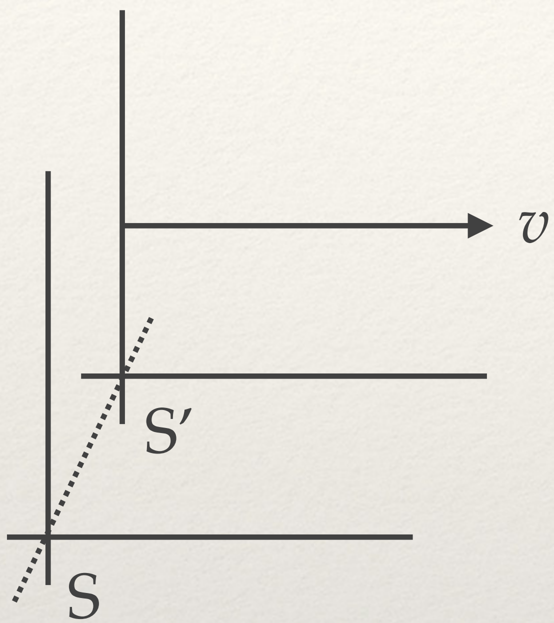
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

- Since these are linear transformations they apply equally well to sums of these "4-vectors".

Lorentz Transformations



$$\beta = \frac{v}{c}$$

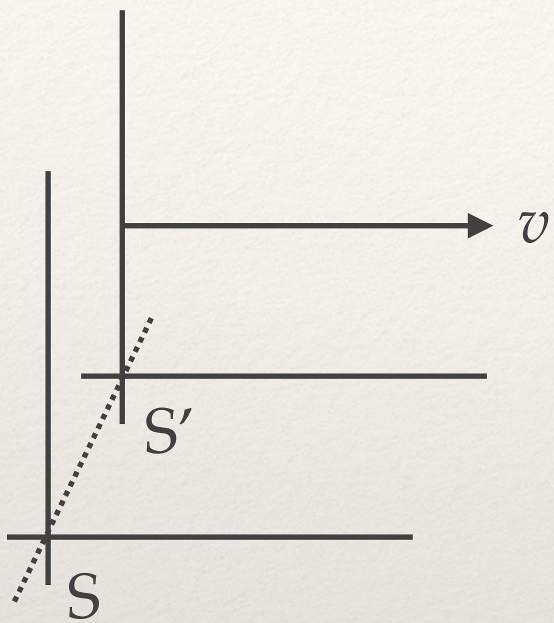
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

- Since these are linear transformations they apply equally well to sums of these “4-vectors”.
- Check energy and momentum in S′ when a particle with mass m is at rest in S (energy=mc^2):

Lorentz Transformations



$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

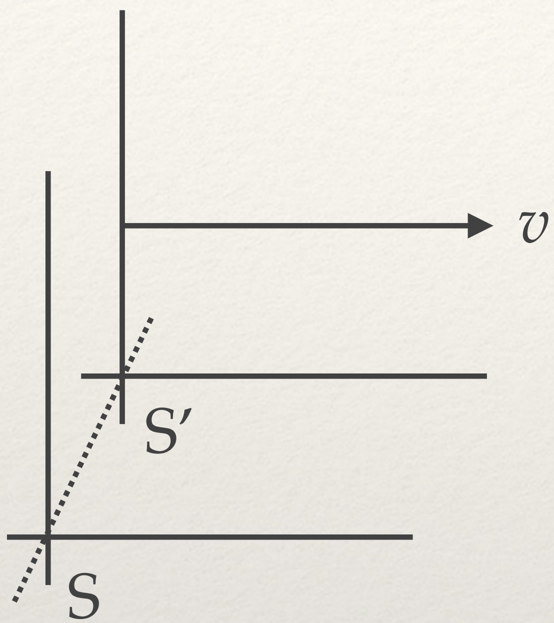
- Since these are linear transformations they apply equally well to sums of these "4-vectors".
- Check energy and momentum in S′ when a particle with mass m is at rest in S (energy=mc^2):

$$p_x{}' = -\gamma\beta E/c = -m\gamma v$$

$$E'/c = \gamma mc^2/c$$

$$E' = m\gamma c^2$$

Lorentz Transformations



$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

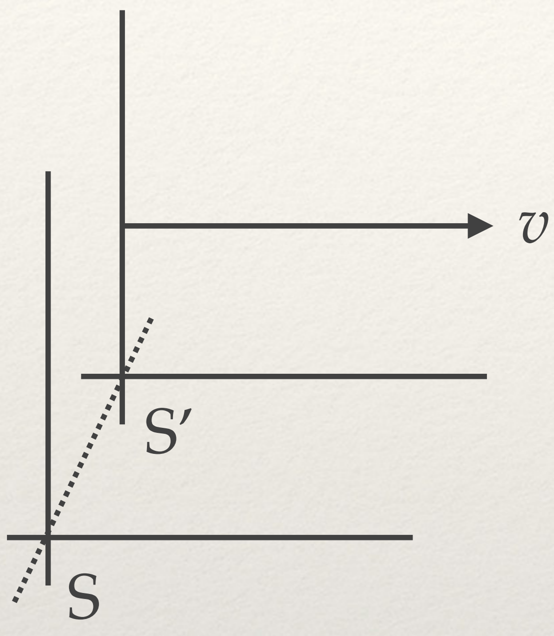
- Since these are linear transformations they apply equally well to sums of these “4-vectors”.
- Check energy and momentum in S′ when a particle with mass m is at rest in S (energy=mc^2):

$$p_x' = -\gamma\beta E/c = -m\gamma v \qquad \text{(particle moves backward in S′)}$$

$$E'/c = \gamma mc^2/c$$

$$E' = m\gamma c^2$$

Lorentz Transformations



$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix}$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}' = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ E/c \end{bmatrix}$$

- Since these are linear transformations they apply equally well to sums of these “4-vectors”.
- Check energy and momentum in S′ when a particle with mass *m* is at rest in S (energy=mc^2):

$$p_x{}' = -\gamma\beta E/c = -m\gamma v \quad ✅ \qquad \text{(particle moves backward in S′)}$$

$$E'/c = \gamma mc^2/c$$

$$E' = m\gamma c^2 \quad ✅$$

4-vector dot product

- Let's define two 4-vectors and a 4-vector dot product.

4-vector dot product

- Let's define two 4-vectors and a 4-vector dot product.

$$\mathbf{A} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ A_0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

4-vector dot product

- Let's define two 4-vectors and a 4-vector dot product.

$$\mathbf{A} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ A_0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &\equiv \vec{a} \cdot \vec{b} - A_0 B_0 \\ &\equiv A^T \eta B \\ &\equiv \left[a_x, a_y, a_z, A_0\right] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix} \end{aligned}$$

Dot product of 4-vectors

- What if we take the dot product in S′ ?

Dot product of 4-vectors

- What if we take the dot product in S′ ?

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

Dot product of 4-vectors

- What if we take the dot product in S′ ?

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

Dot product of 4-vectors

- What if we take the dot product in S′ ?

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma^2(1-\beta^2) & 0 & 0 & -\gamma^2\beta(1-1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma^2\beta(1-1) & 0 & 0 & -\gamma^2(1-\beta^2) \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix} = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

Dot product of 4-vectors

- What if we take the dot product in S′ ?

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}'\cdot\mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma^2(1-\beta^2) & 0 & 0 & -\gamma^2\beta(1-1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma^2\beta(1-1) & 0 & 0 & -\gamma^2(1-\beta^2) \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix} = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\boxed{\mathbf{A}'\cdot\mathbf{B}' = \mathbf{A}\cdot\mathbf{B}}$$

Dot product of 4-vectors

- What if we take the dot product in S′ ?

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\mathbf{A}' \cdot \mathbf{B}' = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} \gamma^2(1-\beta^2) & 0 & 0 & -\gamma^2\beta(1-1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma^2\beta(1-1) & 0 & 0 & -\gamma^2(1-\beta^2) \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix} = \begin{bmatrix} a_x, & a_y, & a_z, & A_0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \\ B_0 \end{bmatrix}$$

$$\boxed{\mathbf{A}' \cdot \mathbf{B}' = \mathbf{A} \cdot \mathbf{B}}$$ Powerful result!

Various dot-products

- Let $\mathbf{X}^T = [\overrightarrow{x}, ct]$, $\mathbf{P}^T = [\overrightarrow{p}, E/c]$

Various dot-products

- Let $\mathbf{X}^T = [\vec{x}, ct]$, $\mathbf{P}^T = [\vec{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \vec{p} \cdot \vec{x} - Et$$

Various dot-products

- Let $\mathbf{X}^T = [\vec{x}, ct]$, $\mathbf{P}^T = [\vec{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \vec{p} \cdot \vec{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

Various dot-products

- Let $\mathbf{X}^T = [\vec{x}, ct]$, $\mathbf{P}^T = [\vec{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \vec{p} \cdot \vec{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

$$\mathbf{P}^2 = |\vec{p}|^2 - E^2/c^2$$

Various dot-products

- Let $\mathbf{X}^T = [\overrightarrow{x}, ct]$, $\mathbf{P}^T = [\overrightarrow{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \overrightarrow{p} \cdot \overrightarrow{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

$$\mathbf{P}^2 = |\overrightarrow{p}|^2 - E^2/c^2$$

(Is this a useful Lorentz invariant? Yes!)

Various dot-products

- Let $\mathbf{X}^T = [\overrightarrow{x}, ct]$, $\mathbf{P}^T = [\overrightarrow{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \overrightarrow{p} \cdot \overrightarrow{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

$$\mathbf{P}^2 = |\overrightarrow{p}|^2 - E^2/c^2$$

(Is this a useful Lorentz invariant? Yes!)

- $\mathbf{P}^2$ is Lorentz-invariant for a single particle, and also for a system of particles. What is it in the rest frame of a single particle, i.e. when $\overrightarrow{p} = 0$?

Various dot-products

- Let $\mathbf{X}^T = [\vec{x}, ct]$, $\mathbf{P}^T = [\vec{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \vec{p} \cdot \vec{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

$$\mathbf{P}^2 = |\vec{p}|^2 - E^2/c^2$$

(Is this a useful Lorentz invariant? Yes!)

- $\mathbf{P}^2$ is Lorentz-invariant for a single particle, and also for a system of particles. What is it in the rest frame of a single particle, i.e. when $\vec{p} = 0$?

$$\mathbf{P}^2 = -m^2$$

Various dot-products

- Let $\mathbf{X}^T = [\vec{x}, ct]$, $\mathbf{P}^T = [\vec{p}, E/c]$

$$\mathbf{X} \cdot \mathbf{P} = \vec{p} \cdot \vec{x} - Et$$

(The factor in the exponential of the free-particle wavefunction is Lorentz-invariant)

$$\mathbf{P}^2 = |\vec{p}|^2 - E^2/c^2$$

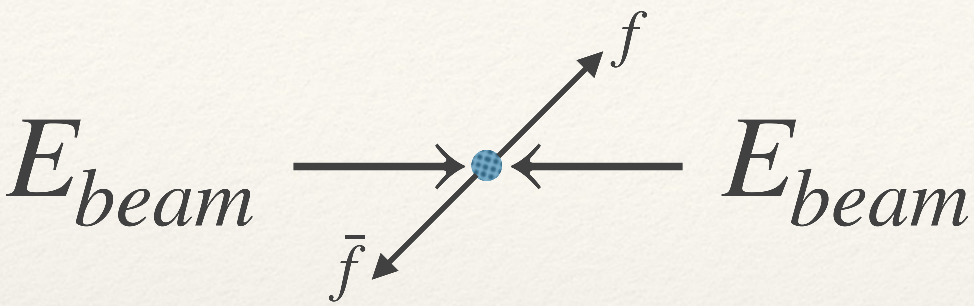
(Is this a useful Lorentz invariant? Yes!)

- $\mathbf{P}^2$ is Lorentz-invariant for a single particle, and also for a system of particles. What is it in the rest frame of a single particle, i.e. when $\vec{p} = 0$?

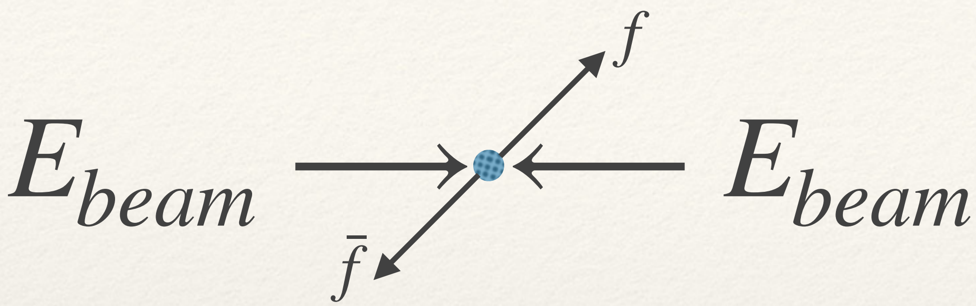
$$\mathbf{P}^2 = -m^2$$

- We identify $W^2 = -\mathbf{P}^2$ as the invariant mass of a single particle, but also the invariant mass of a system of particles that we can calculate in the Lorentz frame with $\sum \vec{p}_i = 0$, i.e. the rest frame of the system of particles.

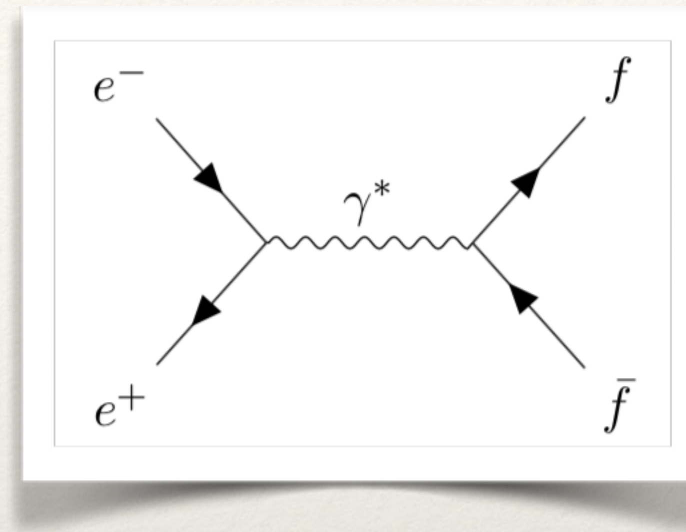
Symmetric collider kinematics



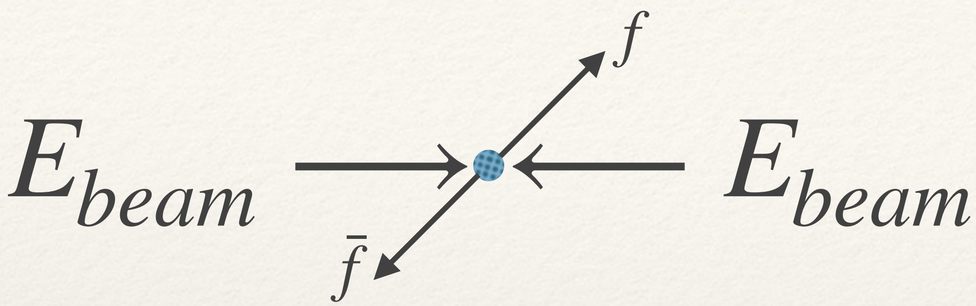
Symmetric collider kinematics

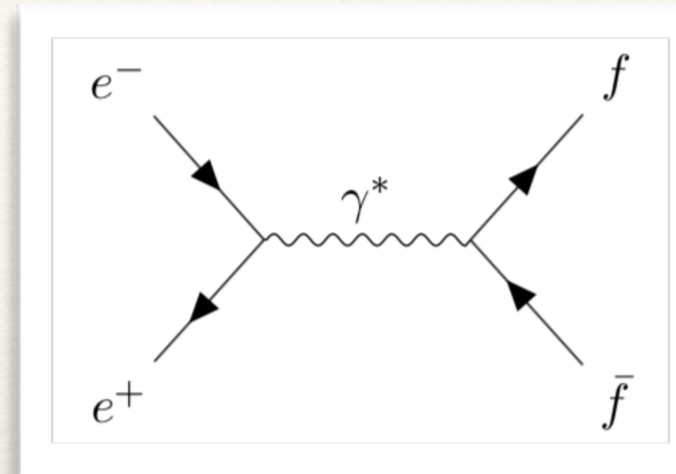


$$e^+e^- \to \gamma^* \to f\bar{f} \text{ (e.g. } \mu^+\mu^-\text{)}$$



Symmetric collider kinematics

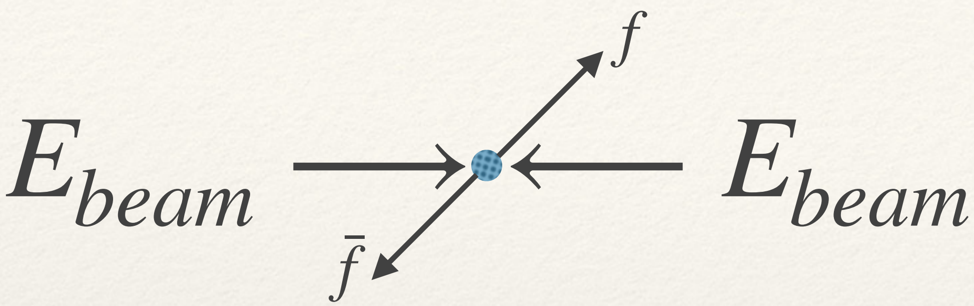




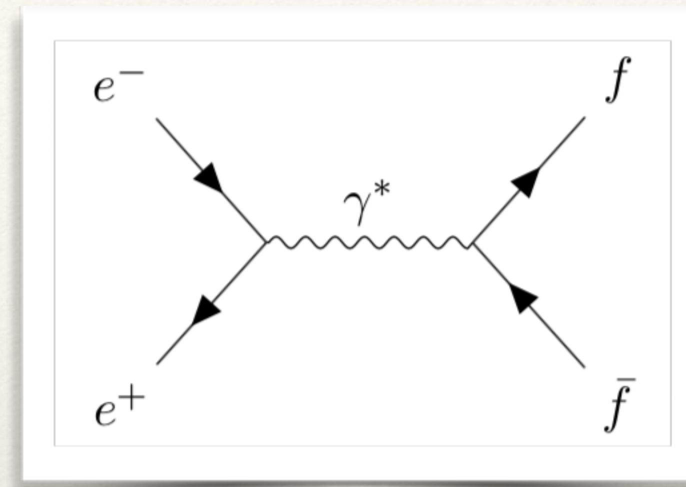
$$e^+e^- \rightarrow \gamma^* \rightarrow f\bar{f} \text{ (e.g. } \mu^+\mu^-\text{)}$$

- What is the invariant mass, or to say it another way, the center of mass energy of the virtual photon?

Symmetric collider kinematics

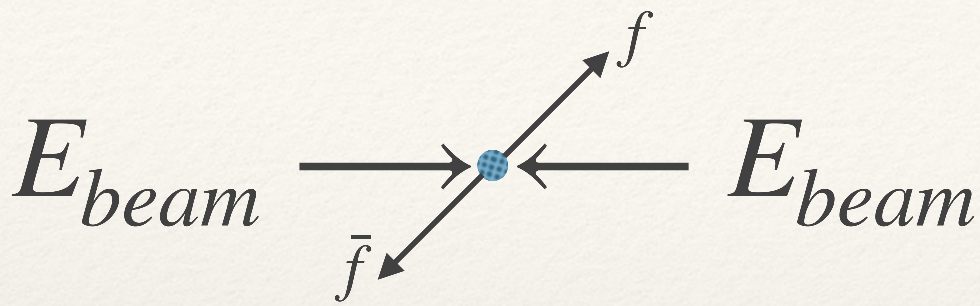


$$e^+e^- \to \gamma^* \to f\bar{f} \text{ (e.g. } \mu^+\mu^-)$$

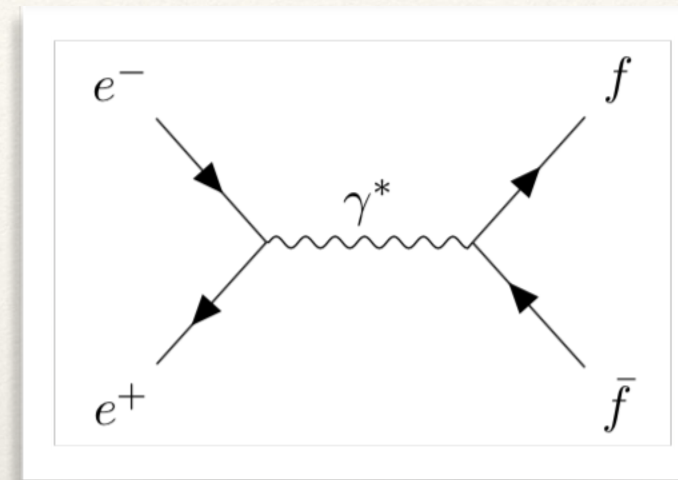


- What is the invariant mass, or to say it another way, the center of mass energy of the virtual photon?
- This laboratory frame is also the center of mass frame $\left(\sum_i \vec{p}_i = 0\right)$

Symmetric collider kinematics



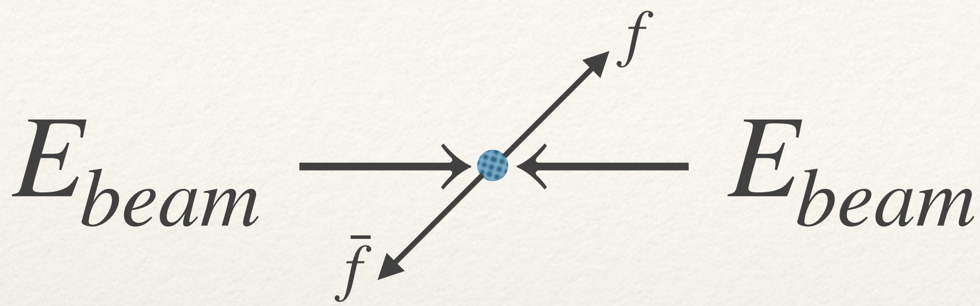
$$e^+e^- \to \gamma^* \to f\bar{f} \text{ (e.g. } \mu^+\mu^-)$$



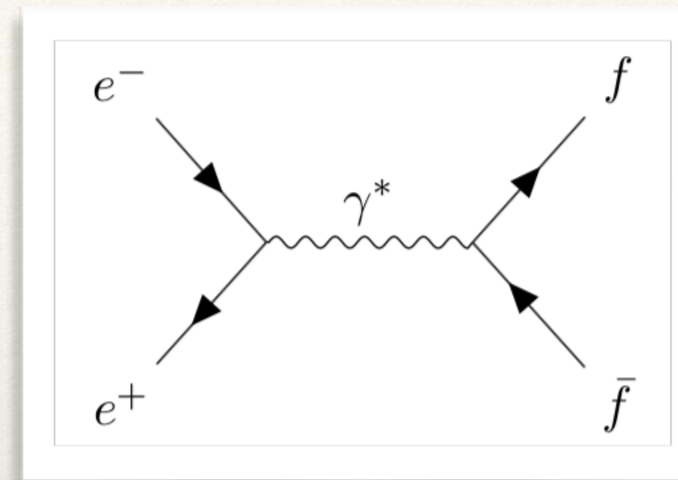
- What is the invariant mass, or to say it another way, the center of mass energy of the virtual photon?
- This laboratory frame is also the center of mass frame $\left(\sum_i \vec{p}_i = 0\right)$

$$"m" \equiv \sqrt{s} = 2E_{beam}, \text{ or } s = 4E_{beam}^2$$

Symmetric collider kinematics



$$e^+e^- \to \gamma^* \to f\bar{f} \text{ (e.g. } \mu^+\mu^-\text{)}$$



- What is the invariant mass, or to say it another way, the center of mass energy of the virtual photon?
- This laboratory frame is also the center of mass frame $\left(\sum_i \vec{p}_i = 0\right)$

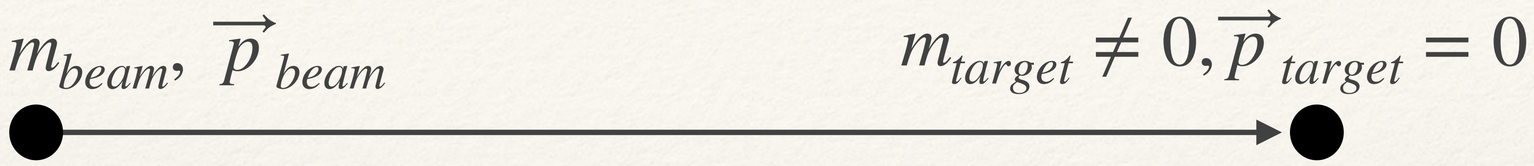
$$\boxed{"m" \equiv \sqrt{s} = 2E_{beam}, \text{ or } s = 4E_{beam}^2}$$

- With enough beam energy and the right couplings we can make a heavy particle at rest and observe its decays (e.g. the Z^0 boson)

Fixed-target kinematics

$m_{beam}, \ \vec{p}_{beam}$ $\quad\quad$ $m_{target} \neq 0, \vec{p}_{target} = 0$

Fixed-target kinematics



- What is the energy in the center of mass?

Fixed-target kinematics

$m_{beam}, \vec{p}_{beam}$ $\quad$ $m_{target} \neq 0, \vec{p}_{target} = 0$

- What is the energy in the center of mass?

$$W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2 = \left(E_{beam} + m_{target}\right)^2 - \vec{p}^{\,2}_{beam}$$

$$= E^2_{beam} + m^2_{target} + 2E_{beam}m_{target} - p^2_{beam}$$

$$= m^2_{beam} + \cancel{p^2_{beam}} + m^2_{target} + 2E_{beam}m_{target} - \cancel{p^2_{beam}}$$

$$= m^2_{beam} + m^2_{target} + 2E_{beam}m_{target}$$

Fixed-target kinematics

$m_{beam}, \vec{p}_{beam}$ $\qquad\qquad$ $m_{target} \neq 0, \vec{p}_{target} = 0$

- What is the energy in the center of mass?

$$W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2 = \left(E_{beam} + m_{target}\right)^2 - \vec{p}^{\,2}_{beam}$$

$$= E^2_{beam} + m^2_{target} + 2E_{beam}m_{target} - p^2_{beam}$$

$$= m^2_{beam} + \cancel{p^2_{beam}} + m^2_{target} + 2E_{beam}m_{target} - \cancel{p^2_{beam}}$$

$$= m^2_{beam} + m^2_{target} + 2E_{beam}m_{target}$$

$$E_{CM} = W = \sqrt{m^2_{beam} + m^2_{target} + 2m_{target}E_{beam}}$$

Antiprotons from proton beam and target

$m_{beam},\ \vec{p}_{beam}$ $\qquad$ $m_{target} \neq 0, \vec{p}_{target} = 0$

$$pp \rightarrow p\bar{p}pp$$

Antiprotons from proton beam and target

$m_{beam}, \vec{p}_{beam}$ $\qquad$ $m_{target} \neq 0, \vec{p}_{target} = 0$

$$pp \rightarrow p\bar{p}pp$$

- Must have minimum energy of 4 proton masses in the center of mass: $E_{CM} \geq 4m_p$

Antiprotons from proton beam and target

$m_{beam}, \overrightarrow{p}_{beam}$ $\qquad$ $m_{target} \neq 0, \overrightarrow{p}_{target} = 0$

$$pp \rightarrow p\bar{p}pp$$

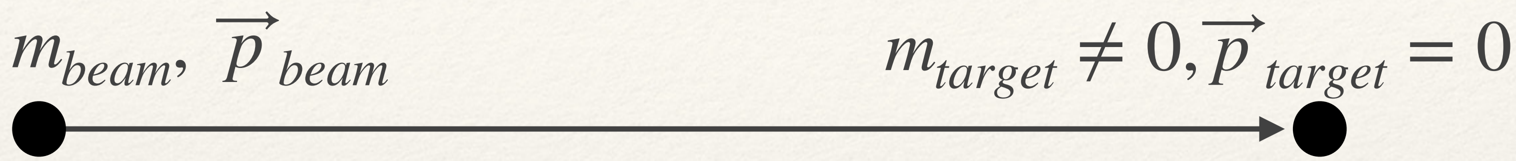
- Must have minimum energy of 4 proton masses in the center of mass: $E_{CM} \geq 4m_p$

$$E_{CM} = \sqrt{m_{beam}^2 + m_{target}^2 + 2m_{target}E_{beam}}$$

$$16m_p^2 = m_p^2 + m_p^2 + 2m_pE_{beam}$$

$$7m_p = E_{beam}$$

Antiprotons from proton beam and target



$$pp \to p\bar{p}pp$$

- Must have minimum energy of 4 proton masses in the center of mass: $E_{CM} \geq 4m_p$

$$E_{CM} = \sqrt{m_{beam}^2 + m_{target}^2 + 2m_{target}E_{beam}}$$

$$16m_p^2 = m_p^2 + m_p^2 + 2m_pE_{beam}$$

$$7m_p = E_{beam}$$

- Need a linear accelerator with proton beam energy above ~7 GeV

Invariant masses of unstable particles

$$W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2$$

Invariant masses of unstable particles

$$W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2$$

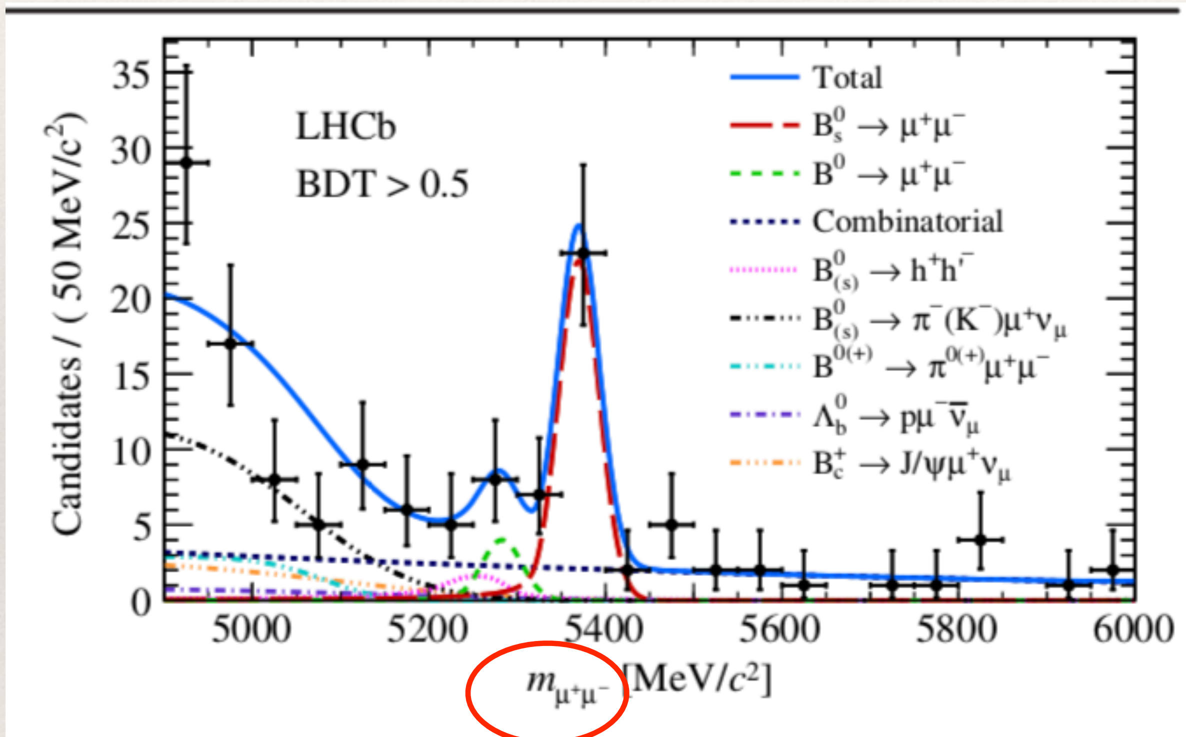


FIG. 1. Mass distribution of the selected $B^0_{(s)} \to \mu^+\mu^-$ candidates (black dots) with BDT > 0.5. The result of the fit is overlaid, and the different components are detailed.

$$B_s^0 \to \mu^+\mu^-$$

Invariant masses of unstable particles

$$W^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2$$

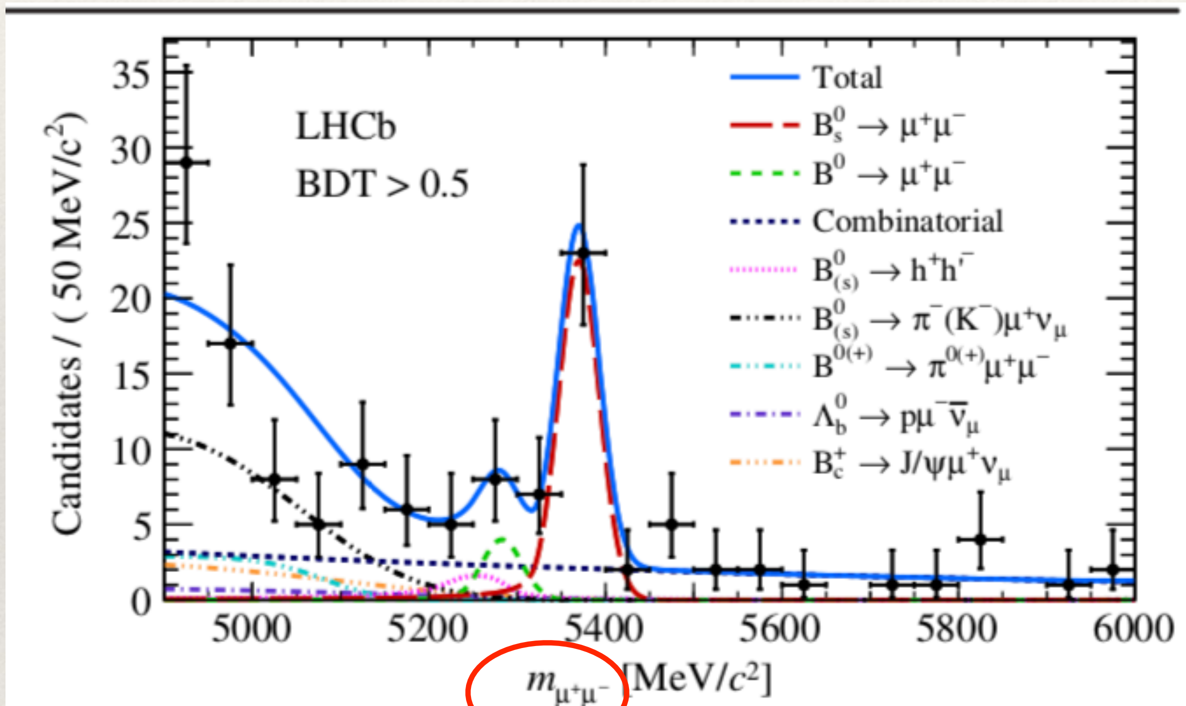


FIG. 1. Mass distribution of the selected $B^0_{(s)} \to \mu^+\mu^-$ candidates (black dots) with BDT > 0.5. The result of the fit is overlaid, and the different components are detailed.

$$B_s^0 \to \mu^+\mu^-$$