FYS3500 - spring 2019

Spacetime Symmetries*

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***Martin and Shaw, Particle Physics, 4th Ed., Chapter 5**

Symmetries and Conserved Quantities

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- Identify symmetries and conservation laws to characterize particles (starting with hadrons)

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- Apply the transformation to $\Psi' = H\Psi$

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- Everything is linear, so also applies to system of particles

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- Small rotations about the (arbitrary) *z*-axis

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- Generalizes to conservation of $\vec{J} = \vec{L} + \vec{S}$

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- We use a similar approach for hadrons, trying to limit the number of different constituents while accounting for many mass states as different orbital and radial excitations of the bound quarks.
- $\vec{J}$, $\vec{L}$, $\vec{S}$ are in general not such great quantum numbers but J, L^2, S^2 are often a good approximation, allowing for spin flips but conserved absolute values.

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Examples: $^1S_0 : J = 0$

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$$\vec{J} = \vec{L} + \vec{S} \;\rightarrow\; J = |L - S], |L - S + 1|, \ldots, |L + S - 1|, |L + S|$$

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- Magnetic moment must come only from the spins of the *n* and *p*: $\mu_d = \mu_n + \mu_p = 2.793 - 1.913 = 0.880$ which is close to the experimental value $\mu_d = 0.857$

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- Lesson is that L is only an *approximate* quantum number for bound states of particles with spin!

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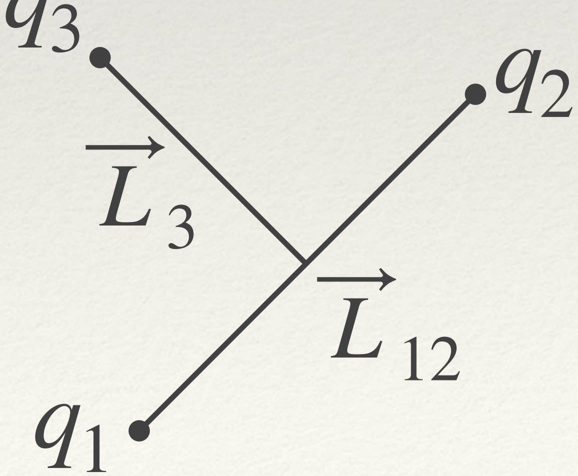
  - Lightest meson states have $L=0$ and lightest baryon states have $L_{12}=L_3=0$.



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  - π, K, and D mesons follow this trend (ρ, K^*, D^* are heavier and have short lifetimes)

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$$\Rightarrow P_a = \pm 1$$

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- Will have to determine intrinsic P_a for each particle

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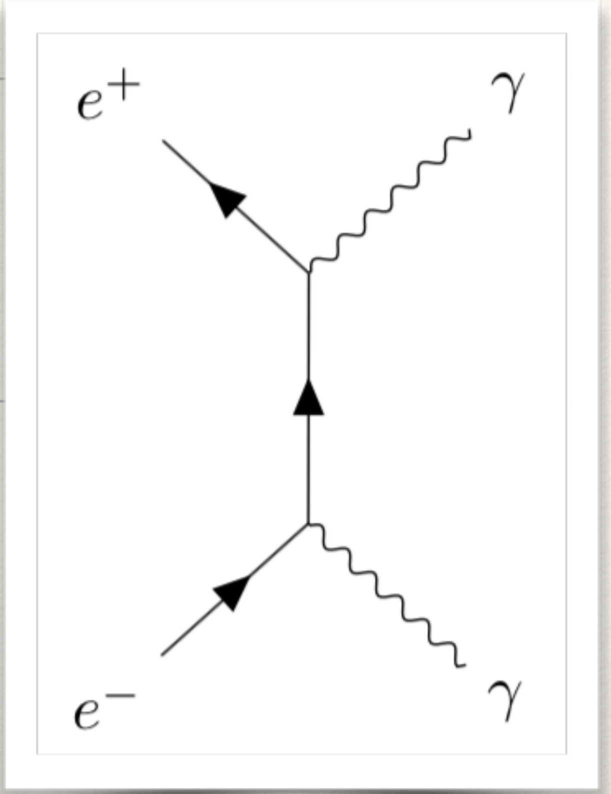
- Since fermions (leptons and quarks) are produced or destroyed in fermion-antifermion pairs, *by convention:*

$$P_f \equiv +1 \quad P_{\bar{f}} \equiv +1$$

Para-positronium



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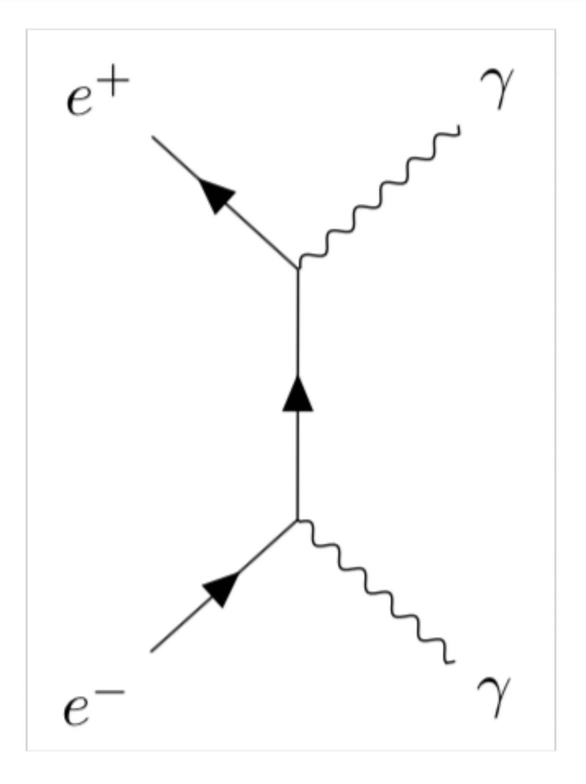

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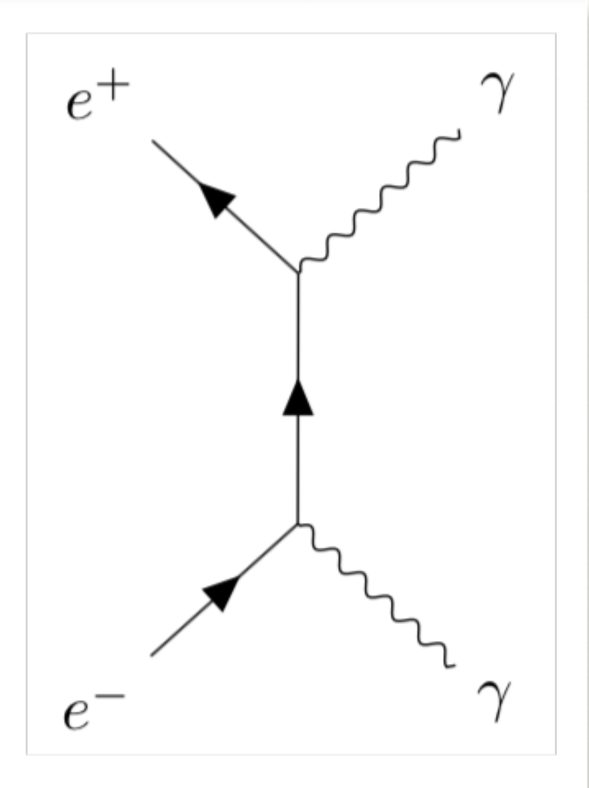

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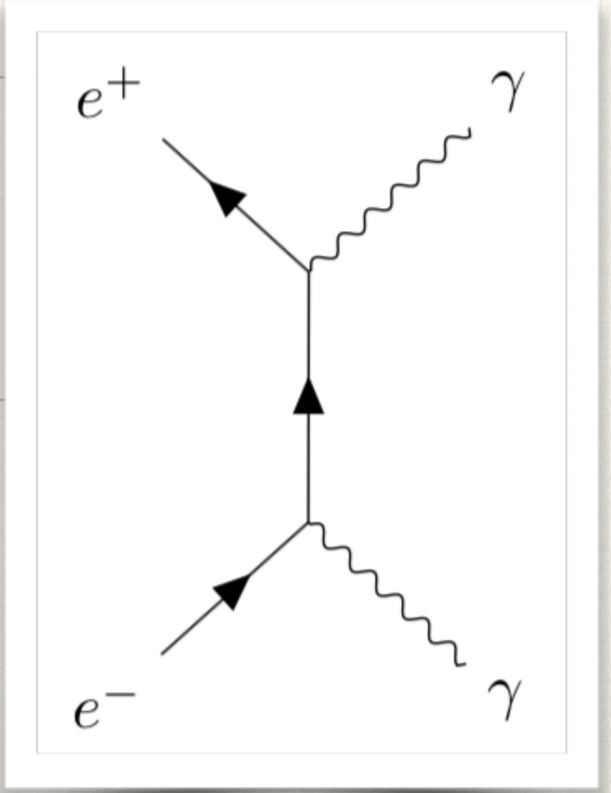

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- l_γ can be determined by measuring the polarization of the two photons, and is consistent with the prediction of 1.

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- Low-mass baryons with $L_{12}=L_3=0$ predicted to have $P=+1$ and corresponding antibaryons $P=-1$

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- while others do not (e.g. π^0, γ)

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- Example: $\hat{C}\,|\,\pi^+\pi^-; L> = (-1)^l\,|\,\pi^+\pi^-; L>$

(particle exchange has same effect as parity trans.)

C-parity for spin-1/2 fermions

$$\hat{C}\,|f\bar{f}; J, L, S> = (-1)(-1)^{S+1}(-1)^{L}\,|f\bar{f}; J, L, S>$$
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- Factor $(-1)^{S+1}$ due to exchange in spin wave functions

$$\uparrow_1 \uparrow_2 \qquad (S = 1, S_z = 1)$$

$$\frac{\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2}{\sqrt{2}} \qquad (S = 1, S_z = 0)$$

$$\downarrow_1 \downarrow_2 \qquad (S = 1, S_z = -1)$$

$$\frac{\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2}{\sqrt{2}} \qquad (S = 0, S_z = 0)$$

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(consistent with arguments one can make about the C-parity of the photon, e.g. M&S 5.4.1)

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$$\hat{C}\left[\eta \to \pi^+(\overrightarrow{p}_1) + \pi^-(\overrightarrow{p}_2) + \pi^0(\overrightarrow{p}_3)\right]$$
$$= \left[\eta \to \pi^-(\overrightarrow{p}_1) + \pi^+(\overrightarrow{p}_2) + \pi^0(\overrightarrow{p}_3)\right]$$