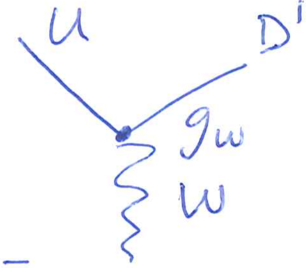
10 ELECTROWEAK UNIFICATION

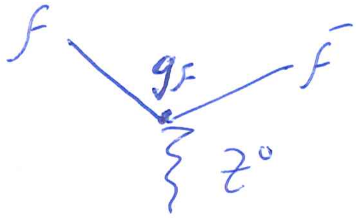
CHARGED CURRENT



$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$

$l = e, \mu, \tau$

NEUTRAL CURRENT



$f = e^-, \mu^-, \tau^-$
ν_e, ν_μ, ν_τ
u, d, s, c, b, t

QUARK SECTOR

$$d'd'Z^0 + s's'Z^0 = \left[(d\cos\theta_c + s\sin\theta_c)^2 + (-d\sin\theta_c + s\cos\theta_c)^2\right] Z^0$$
$$= (dd + ss)\, Z^0$$

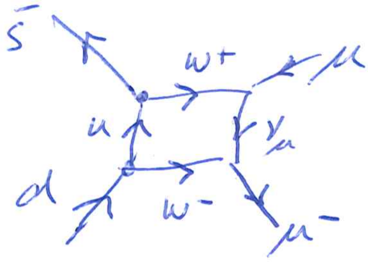
=> CAN USE '-STATES <u>OR NOT</u>

=> $S, C, T, \tilde{B}$ CONSERVED IN Z^0-INTERACTIONS

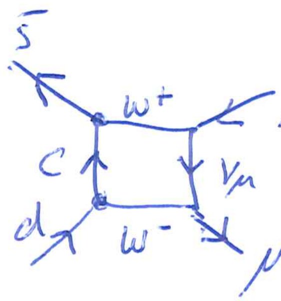
NO FIRST-ORDER FCNC!

$K^0 \to \mu^+\mu^-$

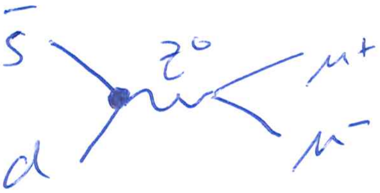
$< 9\cdot 10^{-9}$ BR (2014)



+



+ t

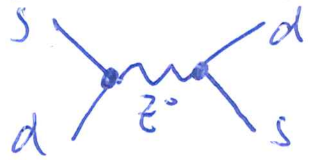


NOT (YET) SEEN

$g_w^4 \cdot \cos\theta_c \cdot \sin\theta_c$ $\quad$ $g_w^4 \cdot \cos\theta_c \cdot (-\sin\theta_c)$

NOT QUITE CANCELLATION DUE TO $m_c \neq m_u$

$K^0 \leftrightarrow \bar{K}^0$ (CHAP. 11)

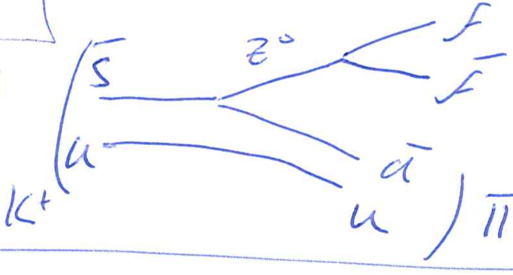


WAY TOO HIGH RATE IF $sdZ^0 \neq 0$.

HOMEWORK: DIAGRAM FOR $K^+ \to \pi^+ \nu_l \bar{\nu}_l$ HINT: (W^+W^-) BOX

$K^+ \to \pi^+ e^+e^-$, $\pi^+ \nu\bar{\nu}$

TINY ~~LIMITS~~ BR's!



BR
e: $3\cdot 10^{-7}$
μ: $9\cdot 10^{-8}$
$\nu\bar{\nu}$: $1.7\cdot 10^{-11}$

GAUGE THEORY (APPENDIX D)

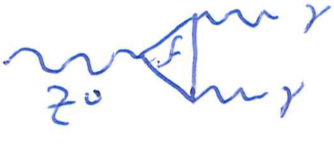
$\Rightarrow B^0, \begin{pmatrix} W_0 \\ W_1 \\ W_2 \end{pmatrix}$ + MIXING $(\theta_w) \Rightarrow$ PHOTON: EM
W^+W^-: CHARGED WEAK
Z^0: NEUTRAL WEAK

TO RECOVER PHOTON $\boxed{e}$ ✓

$$e(2\epsilon_0)^{1/2} = \boxed{g_w}\sin\theta_w = \boxed{g_z}\cos\theta_w$$

$$m_W = \cos\theta_w \cdot m_Z$$

"UNIFICATION CONDITION"



HAS TO VANISH "ANOMALY CONDITION"

COMPLETE GENERATIONS!

$$\sum_{i=1}^{N} \Big(\underbrace{\tfrac{1}{2}(0)^2}_{\nu} - \underbrace{\tfrac{1}{2}(-1)^2}_{l} + \underbrace{\tfrac{1}{2}N_c\left(\tfrac{2}{3}\right)^2}_{U} - \underbrace{\tfrac{1}{2}N_c\left(\tfrac{-1}{3}\right)^2}_{D} \Big) = 0$$

LOW-ENERGY LIMIT OF WEAK INTERACTIONS

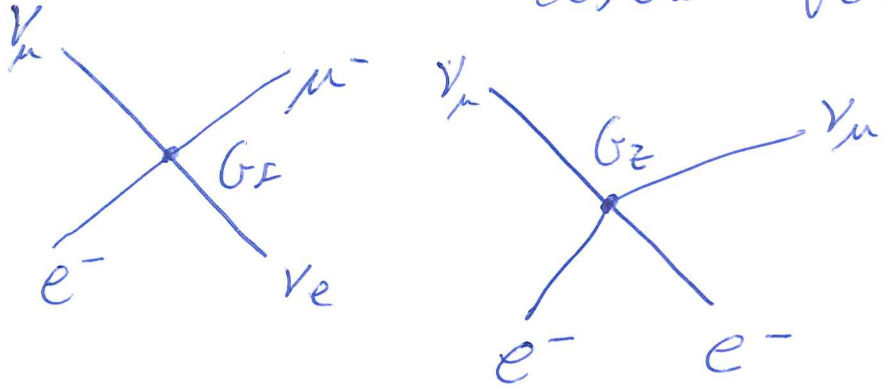
EARLIER $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} \Rightarrow M_W^2 = \frac{\sqrt{2}\, g_W^2}{G_F}$

+ UNI. COND: $M_W^2 = \frac{\sqrt{2}}{G_F}\,\frac{1}{\sin^2\theta_W}\,\frac{e^2}{4\cdot 2\epsilon}\cdot\frac{\pi}{\pi}$

USE $\alpha = \frac{e^2}{4\pi\epsilon_0}$ $\left(\alpha_{EM} = 1/137\right)$

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_W}$$

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_W \cos^2\theta_W}$$

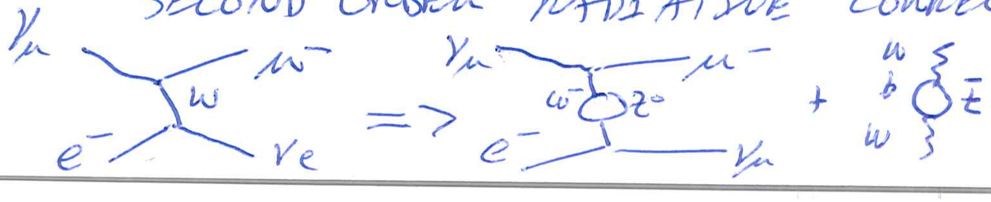


$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}$$

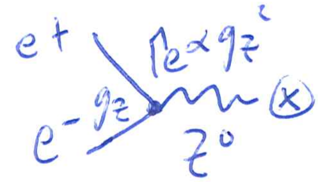
USE $g_W \sin\theta_W = g_Z \cos\theta_W$ AND $M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}$

$\Rightarrow \frac{G_Z}{G_F} = \sin^2\theta_W$, $\sin^2\theta_W \doteq 0.23$

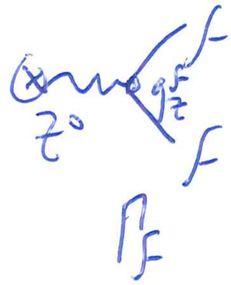
EVERYTHING NEEDED TO PREDICT M_W, M_Z!
(EXPERIMENTS PRECISE ENOUGH THAT SECOND ORDER "RADIATIVE" CORRECTIONS NEEDED)



$$e^+e^- \to Z^0 \to f\bar{f}$$



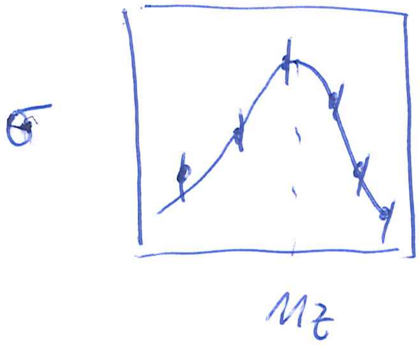
UNLIKE Wff' INTERACTIONS, $Z^0 f\bar{f}$ INTERACTIONS ARE NOT (AS) UNIVERSAL.



U-type quarks
D-type quarks
CHARGED LEPTONS
NEUTRINOS

NO LEPTON - QUARK SYMMETRY
NO UP / DOWN SYMMETRY
STILL LEPTON UNIVERSALITY
SEPARATE UP-TYPE, DOWN-TYPE UNIVERS.

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = \frac{12\pi M_Z^2}{E_{cm}^2} \frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to f\bar{f})}{(E_{cm}^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$



MEASURE HADRONS, LEPTONS VS. E_{cm}

$M_Z = 91.2$ GeV

$\Gamma_Z = 2.5$ GeV

$\Gamma(Z^0 \to q\bar{q}) = 1.744$ GeV (u, d, c, s, b, ~~t~~)

$\Gamma(Z^0 \to l^+l^-) = 0.0840$ GeV

$$\Gamma_Z = \Gamma(Z^0 \to q\bar{q}) + 3\,\Gamma(Z^0 \to l^+l^-) + N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l)$$

$\Rightarrow$ INVISIBLE WIDTH 0.166 GeV $\Rightarrow$ $\underline{N_\nu \cong 3}$

$\Rightarrow$ NO 4-TH GENERATION ν w/ $m_\nu < M_Z/2$
(OF COURSE IT COULD NOT COUPLE TO Z^0 IN SAME WAY).

UNIFICATION AND HIGGS

GAUGE PRINCIPLE:
- PARTICULAR PHASE INVARIANCE OF WAVE FUNCTION.
- INTRODUCE EXTRA TERMS IN EQUATION OF MOTION TO ENSURE PHASE IS UNOBSERVABLE.

- QED CAN BE DERIVED BY

$$\psi(\vec{r},t) \to \psi'(\vec{r},t) = e^{-iqf(\vec{r},t)}\psi(\vec{r},t)$$

$e \to e \cdot$ PHASE-FACTOR $\to e \to e\gamma$

- EW THEORY IN 2x2 ISOSPIN + 1-D (QED-LIKE) HYPERCHARGE

$$\psi(\vec{r},t) \to \psi'(\vec{r},t) = e^{-ig\sum_{i=1}^{3}\hat{I}_i^W f_i(\vec{r},t)} \cdot e^{-ig' Y_W h(\vec{r},t)}\psi(\vec{r},t)$$

$e^- \to \nu_e$, $e^- \to e^-$ $\Rightarrow$ $e^- \to \nu_e W^-$, $\nu_e \to e^- W^+$

[AND $e^- \to e^- B^0$] $\quad$ $e^- \to e^- W^0$, $\nu_e \to \nu_e W^0$

WOULD PREDICT NEUTRAL CURRENTS IN ν-INTERACTIONS FROM ABOVE OBSERVATIONS.

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

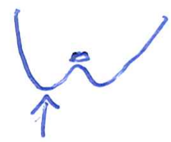
⋮ SOME MODEL

ANOMALY CONDITION

10-5

GAUGE INVARIANCE REQUIRES MASSLESS BOSONS.

SSB – GAUGE INV. EQUATIONS BUT LOWEST ENERGY STATE BREAKS THE SYMMETRY.

- FERROMAGNETISM $\vec{M}$-DIRECTION RANDOM UPON COOLING.
- PENCIL BALANCED ON ITS TIP.
- BALL ON TOP OF MEXICAN HAT 

$$V(\eta) = \mu^2 |\eta|^2 + \lambda |\eta|^4$$

$$\eta \to \eta e^{i\beta}$$

$\boxed{\text{NO PRED of } \mu, \lambda.}$

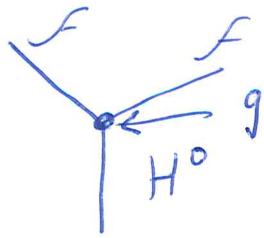
$\lambda > 0$ TO GIVE MINIMUM.

$\mu^2 < 0$ TO GIVE NON-TRIVIAL MINIMUM @ 0

HOMEWORK: SHOW THAT $\beta = 0$, $\eta_0 = \sqrt{-\mu^2/2\lambda}$ IS A SOLN.

<u>CONSEQUENCES</u> (SUMMARY, NO DETAILS) 4-PARAM

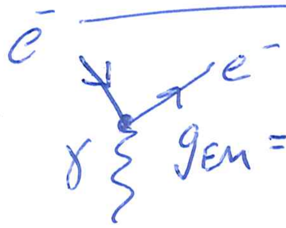
- $W^\pm$, Z^0 MASSES $\neq 0$, $m_\gamma = 0$ (3-COMP)
- H^0 BOSON – EXTRA QUANTUM OF FIELD
- FERMION MASSES FROM INT. W/ H-FIELD.



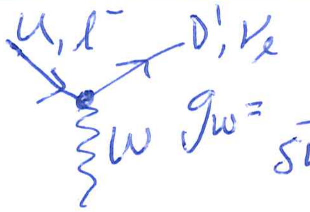
$$g_{Hff} = \sqrt{2}\, g_W \frac{m_f}{m_H},$$ m_f FROM EXPT.

- NO PRED OF m_f BUT PRED. OF H-INTERACTION GIVEN m_f.

SUMMARY OF VERTEX FACTORS FOR ELECTROWEAK INTERACTIONS

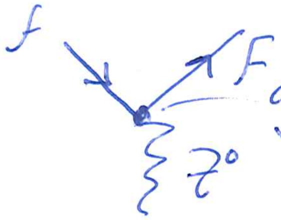


$g_{EM} = e$



$g_W = \dfrac{e}{\sin\theta_W} = O(e)$

NOTE $\left(\dfrac{g_Z}{g_W}\right)^2 = \dfrac{\sin^2\theta_W}{\cos^2\theta_W} \cong 0.26$



$g_Z \cdot \epsilon^f_{L,R}$, $g_Z = \dfrac{e}{\cancel{\sin\theta_W}\cos\theta_W} = O(e)$

f	I^f_3	Q_f	ϵ^f_L
ν_e	$1/2$	0	$1/2$
e^-	$-1/2$	-1	$-\frac{1}{2}+\sin^2\theta_W$
u	$1/2$	$2/3$	$1/2 - 2/3\sin^2\theta_W$
d	$-1/2$	$1/3$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$

$\epsilon^f_L = I^f_3 - Q_f \sin^2\theta_W$

L = "LEFT-HANDED"

FOR $\sin^2\theta_W \cong 0.23$ $\epsilon_L = (0.50, -0.27, 0.35, -0.42)$

ALL $\epsilon_L = O(1)$

f	I_3	Q_f	ϵ_R	ϵ_R
e^-_R	0	-1	$-\sin^2\theta_W$	-0.23
u_R	0	$2/3$	$2/3\sin^2\theta_W$	0.15
d_R	0	$-1/3$	$-1/3\sin^2\theta_W$	-0.08

$\epsilon_R = Q_f \sin^2\theta_W$

R = "RIGHT-HANDED"

ϵ_R FOR ν IS 0!
THE OTHERS SOMEWHAT SMALLER THAN ϵ^f_L.

NOT VERY WRONG TO APPROXIMATE AS $g_Z \cdot \epsilon^f_{L,R}$ AS $\sim 1/2 \cdot g_Z \sim \frac{1}{2}\dfrac{e}{\cancel{\sin\theta_W}\cos\theta_W}$