1) (14P) SHORTER QUESTIONS

FYS 3500
MIDTERM
SPRING 2019

⋮

(2P) b) ANTIPARTICLES?

ALL Q.NUMBERS $Q, B, S, C, \tilde{B}, T$ OPPOSITE.

DIRAC POSTULATED ANTIPARTICLES AS HOLES IN A SEA OF PARTICLES (POSITRONS IN A SEA FILLED WITH ELECTRONS).

- FINDING PARTICLES OF SAME MASS BUT OPPOSITE ELECTRIC CHARGE, FOR EXAMPLE.
- OBSERVING THE ANNIHILATION OF PART.-PART

NOTE! NEUTRAL PARTICLES CAN HAVE DISTINCT ANTI-PARTICLES!! $(n, \bar{n})$, $(D^0, \bar{D}^0)$, $K^0, \bar{K}^0$ etc.

(2P) d) NEUTRON CONSISTS OF udd: $\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$ SO $Q=0$,
BUT MAGNETIC MOMENTS DO NOT CANCEL!
OF CHARGED CONSTITUENTS

BONUS
USING 6.31.b AND 6.30.a (M+S)
$\mu_n = -\frac{2}{3}\mu_p = -1.86\,\mu_N$

⋮

f) Υ MESON IS $b\bar{b}$ BOUND STATE ($Q=0$)
WITH SAME Q.NUMBERS AS PHOTON $J^{PC}=1^{--}$
CAN FIND IT AS ENHANCEMENT IN $e^+e^- \to$ HADRONS
AT THE Υ-MASS OF 9.46 GeV, OR VIA
RECONSTRUCTING ITS INVARIANT MASS

MOST EASILY IN $\Upsilon \to \mu^+\mu^-$ DECAYS.

NO DECAYS TO $B^0\bar{B}^0$, B^+B^- !

4) LIFETIME, DECAY RATE, BRANCHING RATIOS (10P)

$\tau = 10^{-6}\,s$, $B = (50, 35, 15)\%$

WHAT IS DECAY RATE IN PHYSICAL AND NATURAL UNITS?

$\Gamma_{TOT} = \frac{1}{\tau}$, $B_i = \frac{\Gamma_i}{\Gamma_{TOT}}$, $\Gamma_{TOT} = \sum_i B_i$

4P

$\Gamma_i = B_i \cdot \Gamma_{TOT} = \frac{B_i}{\tau}$, $\Gamma_2 = \frac{0.35}{10^{-6}\,s} = 3.5 \cdot 10^{5}\,s^{-1}$ 1P

(PHYSICAL UNITS)

$\Gamma_{NAT} = \Gamma_{PHYS} \cdot \frac{\hbar c}{c} = 3.5 \cdot 10^{5}\,s^{-1} \cdot \frac{197\,MeV \cdot 10^{-15}\,m}{3 \cdot 10^{8}\,m/s}$ 3P

$= 2.3 \cdot 10^{-16}\,MeV$ OR $2.3 \cdot 10^{-10}\,eV$

(NATURAL UNITS)

2P

5.) PROTON - ANTI PROTON ANNIHILATION (6P)

$$p\bar{p} \to \pi^+ \pi^-$$

FIND THE $\pi^+\pi^-$ MOMENTA FOR $p\bar{p}$ ANNIHILATION AT REST.

$p, \bar{p}$ MASSES THE SAME
π^+, π^- MASSES THE SAME
IN CENTER OF MASS $\sum \vec{P}_i = 0$
$\Rightarrow |P_{\pi^+}| = |P_{\pi^-}| \equiv P_{\pi^\pm} \Rightarrow E_{\pi^+} = E_{\pi^-} \equiv E_{\pi^\pm}$ } 2P

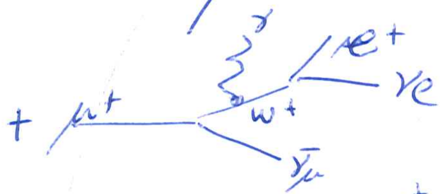
INVARIANT MASS $= W^2 = (E_p + E_{\bar{p}})^2 - (\vec{P}_p + \vec{P}_{\bar{p}})^2$
[COULD ALSO BE DONE W/ CONS. OF ENERGY, MOM.] $= (2m_p)^2 - 0$
$= (2E_{\pi^\pm})^2 - 0$ } 3P

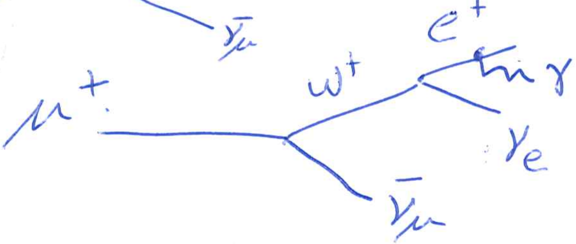
$\Rightarrow E_{\pi^\pm} = m_p$
$|P_{\pi^\pm}| = \sqrt{E_{\pi^\pm}^2 - m_{\pi^\pm}^2}$
$|P_{\pi^\pm}| = \sqrt{m_p^2 - m_{\pi^\pm}^2}$ } 1P

6) NEUTRINO PHYSICS $B(\mu^+ \to e^+\gamma) = ?$
(10p)

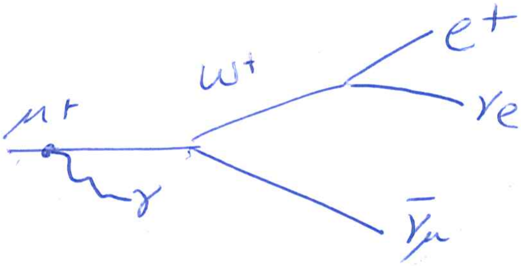
$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ (BRANCHING RATIO ~100%)

$\mu^+ \to \gamma e^+ \nu_e \bar{\nu}_\mu$ (BRANCHING RATIO ~1.4%,
EXPECT $O(\alpha_{em})$ FROM SIMPLE ARGUMENTS.

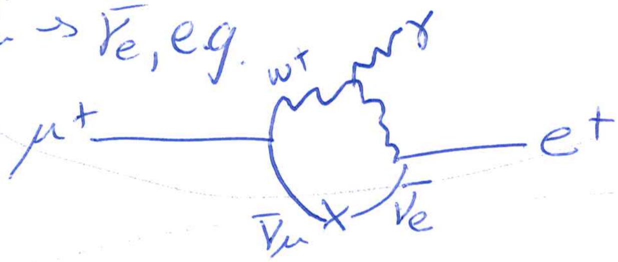




+



EXPECT FURTHER SUPPRESSION FACTOR IF $\nu_e \to \nu_\mu$ OR $\bar{\nu}_\mu \to \bar{\nu}_e$, e.g.



3P

$$B(\mu^+ \to e^+\gamma) = O\left(10^{-2} \cdot P(\nu_\mu \to \nu_e)\right) \quad (2p)$$

USE $P(\nu_\mu \to \nu_e) \cong \sin^2(2\theta_{12}) \sin^2\left(\frac{L}{L_0}\right)$

$\sin^2(2\theta_{12})$

WITH $L << L_0$ AND $\tan^2\theta_{12} = 0.44 \Rightarrow O(1) =$

$$L_0 = \frac{4E}{\Delta m_{12}^2} \cong \frac{4 \cdot m_\mu}{7.5 \cdot 10^{-5}\, eV^2} \qquad m_\mu = 105.7\ MeV$$

$$= 5.6 \cdot 10^{12}\, eV^{-1}$$

1P $L = O\left(\frac{1}{M_W}\right) = 1.3 \cdot 10^{-11}\, eV^{-1}$ $\quad L << L_0 \Rightarrow \sin^2\frac{L}{L_0} \cong \left(\frac{L}{L_0}\right)^2$

$$B(\mu^+ \to e^+\gamma) = O\left(10^{-2} \cdot 1 \cdot \left(\frac{1.3 \cdot 10^{-11}}{5.6 \cdot 10^{12}}\right)^2\right) = 5.4 \cdot 10^{-50}$$

5P

$\sim 5 \cdot 10^{-50}$

7. HIGGS BOSON DECAY WIDTH (10 p)

a) (6 POINTS) $\tau_H = 1.6 \cdot 10^{-22}\,s$

$$\Gamma = \frac{1}{\tau} \cdot \frac{\hbar c}{c} \quad \text{TO GET NATURAL UNITS}$$

$$= \frac{1}{1.6 \cdot 10^{-22}\,s} \cdot \frac{197 \cdot 10^{-15}\,\text{MeV-m}}{3 \cdot 10^{8}\,m/s}$$

$= 4.1$ MeV, WHICH IS 10^3 SMALLER THAN STATED $\Gamma = 4.1$ GeV.

(4 POINTS)

b) THE SIMPLEST WAY TO INCREASE THE WIDTH OF THE HIGGS BOSON BY A FACTOR 10^3 WOULD BE TO INTRODUCE A NEW DECAY MODE WITH A VERY LARGE DECAY RATE Γ'!

$$\Gamma = \Gamma_0 + \Gamma'$$

Γ_0 ← SM DECAYS; Γ' ← NEW DECAY MODE WITH LARGE WIDTH

THE NEW DECAY MODE SHOULD EITHER BE SOMETHING WE HAVEN'T LOOKED FOR EARLIER (BUT IS DISTINCT FROM THINGS ALREADY MEASURED/OBSERVED) OR SOMETHING THAT ISN'T EASY TO OBSERVE.

8) QUANTUM NUMBERS (10P)

a) (2P) PARITY: SPACE INVERSION

$$\vec{x} \rightarrow -\vec{x}$$

EIGENSTATES OF PARITY ± 1

PARITY CONSERVED IF $H(-\vec{r}_1, -\vec{r}_2 \ldots) = H(\vec{r}_1, \vec{r}_2, \ldots)$

C: CHARGE CONJUGATION

PARTICLES <=> ANTI PARTICLES

EIGENSTATES HAVE $C = \pm 1$ (MUST HAVE $Q=0$)

PARTICLES THAT ~~ARE~~ ARE THEIR OWN ANTIPARTICLE HAVE $C = \pm 1$.

PARTICLE-ANTIPARTICLE PAIRS CAN ALSO BE EIGENSTATES.

b) (3P) LIGHTEST SPIN-0 MESONS ($q_i \bar{q}_i$)

$$\hat{C}\,|f\bar{f}, J, L, S\rangle = (-1)^{L+S}\,|f\bar{f}, J, L, S\rangle$$

LIGHTEST SPIN-0 HAS $J = L = S = 0$

$$\hat{C} = 1$$

$$P = (-1)^{L+1} = -1$$

SO LIGHTEST MESONS (SPIN-0) HAVE $J^{PC} = 0^{-+}$

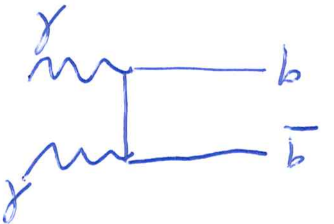
9) ALLOWED, SUPPRESSED, FORBIDDEN PROCESSES (20 p)

$\nu_e e^- \to \nu_\mu \mu^-$

(2 p) 1) $L_e = 2 \to L_e = 0$ AND $L_\mu = 0 \to 2$

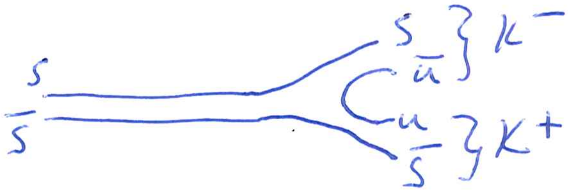
SO <u>FORBIDDEN</u> (DISREGARDING ν-MIXING, WHICH IS SUPER-TINY)

(2 p) 2) $\gamma\gamma \to b\bar{b}$



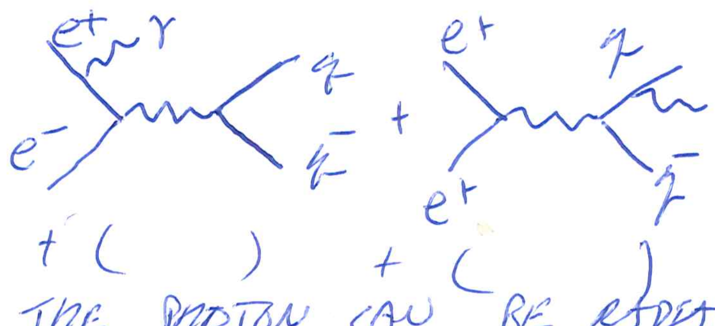
ELECTROMAGNETIC INTERACTION

3) $\phi \to K^+K^-$



COULD BE EM OR STRONG, BUT STRONG WILL DOMINATE. THE K^+K^- BRANCHING FRACTION IS ~50% AND THE WIDTH IS 4.4 MeV CORRESPONDING TO A LIFETIME OF $\sim 1.5 \cdot 10^{-22}$ s, WHICH IS TYPICAL FOR STRONG INTERACTION DECAYS OF HADRONS.

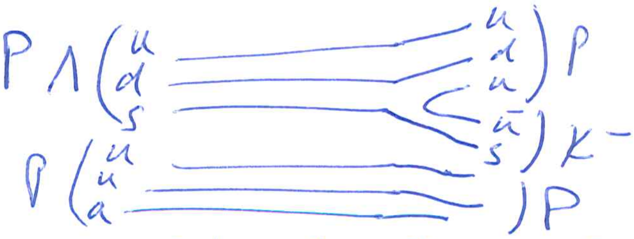
4) $e^+e^- \to q\bar{q}\gamma$



THE PHOTON CAN BE RADIATED FROM ANY OF THE 4 FERMION LINES. EM. INTERACTION

BONUS: THE EXCHANGE PARTICLE COULD ALSO BE A Z^0

5) $\Lambda p \to K^- p p$

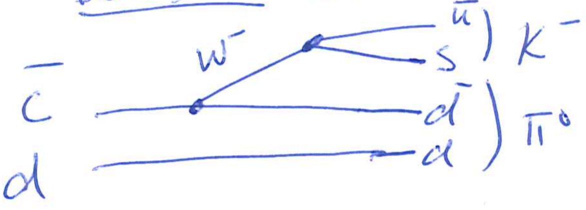


ALLOWED REACTION, MOSTLY LIKELY STRONG.

9) - CONT

6) $D^- \to K^- \pi^0$ [MORE THAN A BIT UNFAIR, BASED ON LATER CHAPTER IN M+S]

DOUBLY CABIBBO SUPPRESSED WEAK INTERACTION!



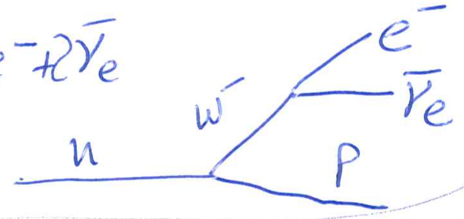
D^- LIFETIME IS $\sim 10^{-12}$ s
TYPICAL FOR WEAK INTERACTION.

BRANCHING RATIO IS $\sim 2 \cdot 10^{-4}$
CONSISTENT W/ WEAK AND C^2-SUPPRESSED.

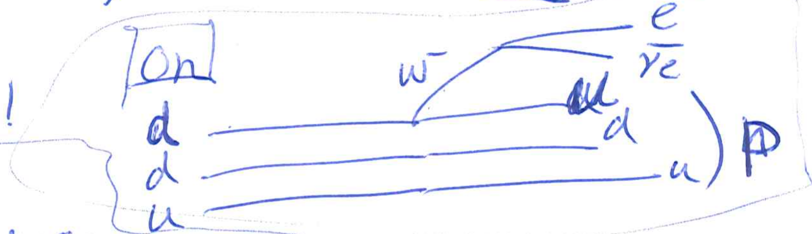
7) $\mu^+\mu^- \to \gamma$: CAN'T GO IN FREE SPACE, WOULD GIVE $m_\gamma > 0$!

8) ${}^{82}_{34}Se \to {}^{82}_{36}Kr + 2e^- + 2\bar{\nu}_e$

$2\times(n \to p + e^- + \bar{\nu}_e)$



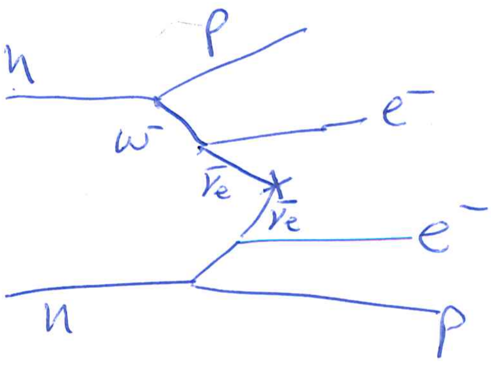
THE PROTONS STAY IN THE NUCLEUS!



10^{20} YR HALF-LIFE CONSISTENT W/ DOUBLY WEAK SUPPRESSED PROCESS.

9) ${}^{76}_{32}Ge \to {}^{76}_{34}Se + 2e^-$

NEUTRINOLESS DOUBLE BETA DECAY.
NOT POSSIBLE UNLESS $\nu_e = \bar{\nu}_e$



9 - CONT

e) (ii) PROCESSES 2 AND 4 COULD BE USED TO TEST QUARK CHARGES.

- THE RATE OF $\gamma\gamma \to \mu^+\mu^-$ AND $\gamma\gamma \to b\bar{b}$ WOULD BE RELATED BY $N_c \cdot q_b^4$, WHERE N_c = # COLORS.
- COMPARING THE RATE OF $e^+e^- \to q\bar{q}$ TO $e^+e^- \to q\bar{q}\gamma$, AND ~~STRACTING~~ SUBTRACTING THE CONTRIBUTION OF RADIATION FROM THE INITIAL e^+, e^-, WOULD GIVE A RATIO OF $\alpha_{em} \cdot q_b^2$; THE N_c FACTOR WOULD CANCEL.

~~f~~) (1p) SEPARATE (8) AND (9) BY MEASURING THE ENERGY SPECTRUM OF THE $2e^-$. WE WOULD LOOK ~~AT~~ FOR A LITTLE EXCESS AT THE MAXIMUM ELECTRON ENERGY.