FYS3520 - Problem set 1

Spring term 2019

Problem 1: Commutator relations – Discussion in class

a) Derive following commutator relations. What is their significance?

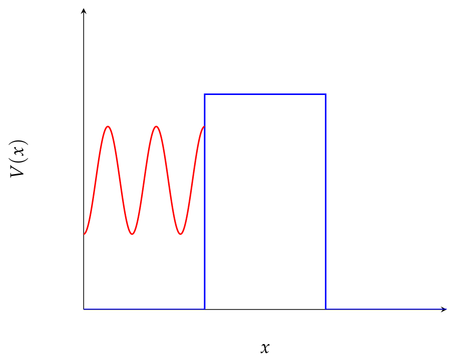
$$[\widehat{L}_x, \widehat{L}_y] = i\hbar\widehat{L}_z, \qquad [\widehat{L}_x, \widehat{y}] = i\hbar\widehat{z}, \qquad [\widehat{L}_x, \widehat{p_y}] = i\hbar\widehat{p}_z,$$
$$[\widehat{L}_x, \widehat{x}] = [\widehat{L}_x, \widehat{p}_x] = [\widehat{L_x}, \widehat{L}^2] = [\widehat{L}_x, \widehat{r^2}] = [\widehat{L}_x, \widehat{p}^2] = 0$$

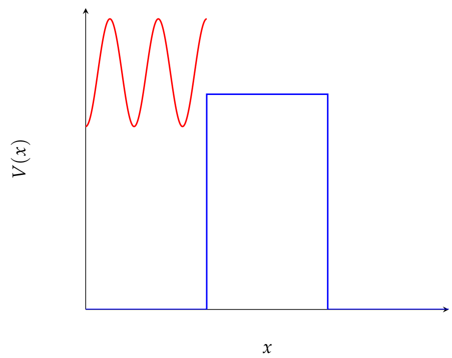
Hint: You may use $\mathbf{L} = \mathbf{r} \times \mathbf{p} \to \widehat{\mathbf{L}} = \widehat{\mathbf{r}} \times \widehat{\mathbf{p}}$ and $[\widehat{r}_i, \widehat{p}_j] = i\hbar\delta_{ij}$

b) Given a system with the angular momentum $L = 0$ ($L = 2$) and spin $S = 1/2$, write down the states and their degeneracy in spectroscopic notation: ${}^{2S+1}L_J$.

Problem 2: Potential barrier – Discussion in class

We want to get a qualitative understanding of the situation where an incoming free particle from the left with energy E hits on a potential barrier $V(x)_{II} > 0$. The particle can be taken as wave-package produced by a superposition of plain waves. In the following graphs, two such cases are plotted for a the real part of the function, $Re(\psi(x)) = \cos(k_I x)$. Sketch how the wave will continue in the different regions and compare the situation to a classical system.





Problem 3: Probability density – Discussion in class

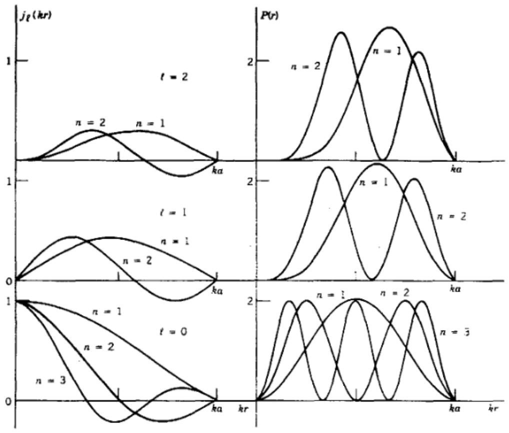
In a simple model, the nuclear potential can be assumed to be an **infinite spherical well**. The solutions of the Schrödinger equation are then given by product of the radial function $R(kr)$, which are proportional to the Bessel functions $j_l(kr)$, and the spherical harmonics $Y_l^m(\vartheta, \varphi)$.

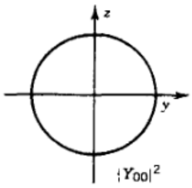
a) What is the meaning of the different symbols? What quantum numbers does the energy of the states depend on? What is there degeneracy?

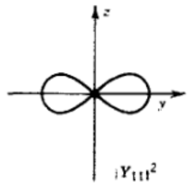
b) What is the relation between the quantum numbers?

c) Sketch the probability density $|\psi_{n,l,m}|^2$ for i) n=0, l=0, m=0 and ii) n=2, l=1, m=1.

d) What is the parity of these states?







Problem 4

Use the notation ${}^{A}_{Z}\mathrm{X}_{N}$ with mass number A, element number Z and neutron number N.

a) How many neutrons does ^{232}Th and ^{235}U have?

b) List all **stable** nuclei with mass number 20.

c) List all **stable** nuclei with element number 82.

d) List all **stable** polonium (Po) nuclei.

e) Write down all the **stable** Zirconium (Zr) isotopes.

Hint: Charts of most known nuclides can be found at `http://www.nndc.bnl.gov/chart/chartNuc.jsp` or at `https://www-nds.iaea.org/livechart`.

Problem 5

Of the following nuclei: ^{16}O, ^{19}Ne ^{18}F, ^{15}N, ^{16}C, ^{20}O, ^{16}Ne, ^{14}C, ^{20}N and ^{20}Ne.

a) Write out the full ${}^{A}_{Z}\mathrm{X}_{N}$ notation for all the listed nuclei.

b) List the nuclei that are **isotopes**.

c) List the nuclei that are **isotones**.

d) List the nuclei that are **isobars**.

e) What is an **isomer**?

Problem 6

What is the mass of one ^{12}C and ^{239}Pu nucleus in

a) Atomic mass units(AMU) [u]?

b) Units of [MeV/c^2]?

c) In [kg]?

Hint: There are several databases containing properties of known nuclei. The Atomic Mass Data Center(AMDC) have lists of masses of most of the known nuclides avalible as a text file at `https://www-nds.iaea.org/amdc/` or as an interactive table at `https://www-nds.iaea.org/livechart`.

Problem 7: Cats in 1D

The **one-dimensional** time-independent Schrödinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

a) Derive this from the time dependent Schrödinger equation for a time-independent potential $V(x,t) = V(x)$.

b) Assume that the potential is a infinite well, ie. infinite at $x < 0$ and $x > a$ and zero in between ($0 < x < a$). Show that the eigenstates are given by

$$\psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}, \quad n = 1,2,3,\ldots$$

and find the energy E_n for each state.

c) Assume that we have a particle in the super-position: $\Psi(x) = A(\psi_1(x) + \psi_3(x))$. Find the normalization factor A.

d) Find the average energy of $\Psi(x)$. What is the variance?

e) What is the propability of finding the particle between $\frac{a}{4} \leq x \leq \frac{3a}{4}$?

Problem 8: Binding energy: a start (after 3rd lecture)

a) The mass of ^{27}Al and ^{235}U has been measured to be 26.9815 u and 235.0439 u respectively. Use this to estimate the binding energy of the two.

b) Find the binding energy of the two nuclei using the semi empirical mass formula. How well does it agree with the result in a)/what are possible reasons for the deviations?
(Use $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV).

Problem 9 – A nice little extra

The potential of a simple one-dimensional harmonic oscillator in one dimension can be described by $V(x) = \frac{1}{2}m\omega^2 x^2$.

a) Find the energy of the number state $|n\rangle$ using ladder operators:

$$\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} + i\hat{p})$$
$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} - i\hat{p})$$

Remember: $\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$, $\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$, and $[\hat{a}, \hat{a}^\dagger] = 1$.

b) What two particle states with fermions are possible with the three first states (ignoring spin)?

c) Give the degeneracy for 3D symmetric HO, so $\omega_x = \omega_y = \omega_z$, with $E = \frac{7}{2}\hbar\omega$.