FYS3500 - Problem set 7

Spring term 2019

Updated: Problem 6, correected eq 1 & 2
Updated2: Problem 6, hint on $t < 0$

Problem 1 – in class

a) What types of nuclear decay are there? Characterize the different types of decay.

b) What is the decay law and how do we derive it?

c) What is the relation between the width Γ of a state and its lifetime? How does Γ effect the notion of discrete states?

d) Explain what the branching ratios is and how this we may measure them.

e) Assume you want to measure the half-live against γ-decay in an experiment? What set-up could you choose? What types of uncertainties will be associated to the result?

Problem 2 – in class

a) What is the standard picture for α-decay? Why is the α particle so prominent? Are there also other decay modes, e.g. ^{3}He, ^{9}Be or ^{14}C or ^{24}Na?

b) What is the relative biological effect of the different kinds of radiation? Why can I hold an α source in my hand without necessarily suffering a lot of radiation damage?

Problem 3 General aspects of the α-decay

1. What are the requirements for spontaneous decay from a parent to daughter nucleus? What terms in the liquid drop model lead to α-decay of a proton (/neutron) rich nucleus?
2. What determines the α-decay probability? Why is the cluster decay usually strongly suppressed?
3. How does the energy spectrum of the α-particle from an α-decay look like? Why does the energy of the α-particle deviate from the Q-value of the decay?

Problem 4 Radioactive Sources

Three radioactive sources each have activities of 1 μCi at t=0. Their half-lives are, respectively, 1.0 s, 1.0 h, and 1.0 d.

a) How many radioactive nuclei are present at $t = 0$ in each source?

b) How many nuclei of each source decay between $t = 0$ and $t = 1$ s

c) How many decay between $t = 0$ and $t = 1$ h?

Problem 5 Nuclear Archeology and nuclear physics in archeology

^{14}C is used to determine the age of fossils and other organic materials. The idea is that as long an organism is alive, it constantly is exchanging carbon with its environment (eating and excreting) and so the isotopic composition of the organism matches that of the atmosphere. Once the organism dies, this exchange stops, and the ^{14}C trapped in the system start ^{14}C/^{12}C ratio was the same in the past as it is today (which is almost true, but hang on for a surprise . . .), then if we see less ^{14}C it must be because this isotope has decayed (^{12}C is stable).

a) What determines the time-scales that we can use this method on?

b) What gave a sharp rise to the fraction of ^{14}C in the atmosphere from about the 1960's.

If we want to look at events that take much longer than 5000 years, it's useful to look for radioactive decays that have much longer half lives. If you poke around the periodic table, you find that heavy elements often have radioactive isotopes with half lives measured in billions of years. Let's focus on Naturally occurring uranium is a mixture of the 238 U (99.28 %) and 235 U (0.72 %) isotopes.

c) How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? How do you interpret this result?

d) How much of the ^{238}U has decayed since the formation of the Earth's crust 2.5×10^9 y ago?

Problem 6 Bonus: Lifetime and natural line widths

Proof that the relationship between the lifetime τ of a decaying state and its (natural) line width Γ is given by $\Gamma\tau = \hbar$. Remember that Γ is the full width at half maximum (FWHM).

If we observe an exponential decay of the initial state i, we may assume following form for the wave function:

$$|\psi(t)|^2 = \exp(-\Gamma t) \rightarrow \psi(t) \propto \exp(-\Gamma t/2) \quad (1)$$

The time dependence would be given by the Schroedinger equation, as usual, by:

$$\psi(t) \propto \psi(t = 0) \exp(-iE_i t/\hbar) \propto \exp(-\Gamma t/2) \exp(-iE_i t/\hbar) \quad (2)$$

For normalization we assume further that $\psi(t < 0) = 0$.

a) Show that

$$\psi(E) \propto \frac{1}{i(E - E_s) - \Gamma/2},$$

and that the energy distribution is given by a Lorentzian function:

$$|\psi(E)|^2 \propto \frac{1}{(E - E_s)^2 + (\Gamma/2)^2}.$$

Hint: You should use Eq. (2) as a starting point. The time and energy (more exactly frequency) domain are related by a Fourier transformation, just as space and momentum.

b) Plot the resonance for the γ line of the ^{60}Co source (which is the ^{60}Ni nucleus) given on the last problem set: $E_s = 1333 keV$, $t_{1/2} = 0.7$ ps. You may verify that Γ is the half-width of the distribution.

You may read more on that subject, especially also on the generalization to an actual nucleus in one of the classical monographs in the field, Blatt and Weisskopf, *Theoretical Nuclear Physics* (1952). p. 412ff.