FYS3500 - Problem set 9

Spring term 2019

Problem 1 – in class

a) Did you calculate all problems from the previous sets? Any questions?

b) Start with the problem below; after some examples of (a), focus especially (b to d).

Problem 2 γ-decay and Weisskopf units

The reduced transition probabilities are expressed in $e^2\text{fm}^{2L}$ for EL multipoles and in $\mu_N^2\text{fm}^{2L-12}$ for ML multipoles, see eg. Krane.

a) For the following transitions between levels, give all permitted γ-ray multipoles and indicate which multipole might be the most intense in the emitted radiation.
a) $\frac{9}{2}^- \to \frac{7}{2}^+$ b) $\frac{1}{2}^- \to \frac{7}{2}^-$ c) $1^- \to 2^+$ d) $0^+ \to 0^+$ e) $3^+ \to 3^+$ e) $4^+ \to 2^+$
f) $\frac{11}{2}^- \to \frac{3}{2}^+$

b) A nucleus has the following sequence of states beginning with the ground state: $\frac{3}{2}^-$, $\frac{7}{2}^-$, $\frac{5}{2}^+$, $\frac{1}{2}^-$ and $\frac{3}{2}^-$ Draw a level scheme showing the intense γ transitions likely to be emitted and indicate their multipole assignment. Which of the transitions would you expect to be have the smallest chance to happen?

c) Which γ would you expect to be emitted in coincidence with each other (your timing resolution may be something like 50ns). How would a low-lying excited state with high spin behave?

d) Assume now, that the daughter nucleus is populated by β-decay with a Q-value of 7 MeV from the $I = 3/2^-$ gs of a mother nucleus. Sketch the electron spectrum. Hint: Is this enough information? What else do you potentially need? If in class: Ask for the information. If at home: Make assumptions where necessary.

e) For a light nucleus ($A = 10$), compute the ratio of the emission probabilities for electric quadrupole (E2) and magnetic dipole (M1) radiation according to the Weisskopf estimates. Consider all possible choices for the parities of the initial and final states.

f) Compare this to the ratio calculated for a heavy nucleus (A = 200).

Problem 3 Bonus: Medical isotope production

Technetium-99m ($^{99m}_{43}\text{Tc}_{56}$) is a metastable nuclear isomer of ^{99}Tc (itself an isotope of Tc) that is used in tens of millions of medical diagnostic procedures annually, making it the most commonly used medical radioisotope.[1]

The technetium ^{99m}Tc (excited state of ^{99}Tc) is a γ-emitter used in nuclear medicine to for example detect cervical tumors with a γ-camera.[2]

$$^{99m}\text{Tc} \;\to\; ^{99}\text{Tc} + \gamma$$

^{99m}Tc is a byproduct of the β^--decay of ^{99}Mo:

$$^{99}\text{Mo} \;\to\; ^{99m}\text{Tc} + e^- + \bar{\nu}_e$$

[1] Tc-99m: `https://en.wikipedia.org/wiki/Technetium-99m`

[2] Based on Exercise 2.1 in `https://lpsc.in2p3.fr/schien/PHY113a/TD_radio_en_2011-2012.pdf`

A simplified decay scheme is shown in Figure 1

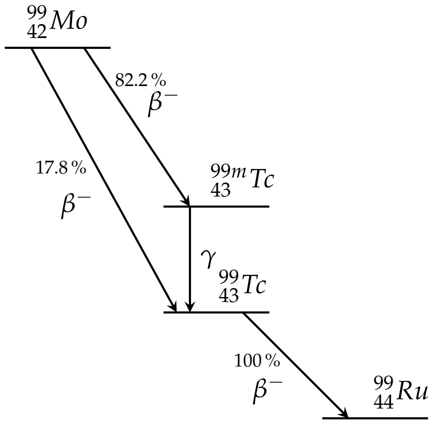

Figure 1: Simplified decay scheme of ^{99}Mo.

^{99}Mo can be produced for example in a cyclotron. Let's assume that the initial activity of a ^{99}Mo source is $A(0) = 8.5$ Ci and that source does not contain any Tc initially.

a) Plot the number of nuclei ($N_x(t)$ vs t) and activity ($\mathcal{A}_x(t)$ vs t) for ^{99}Mo, $^{99\text{m}}$Tc, ^{99}Tc and ^{99}Ru for the first 300 hours.
Hint: You may want to find informations such as half-lives etc. from `http://www.nndc.bnl.gov/chart/` or `https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`. Start by calculating the number of ^{99}Mo nuclei present at $t = 0$. You may either derive the equations for the radioactive decay series, or look them up. In any case, remember to take into account the branching in the decay of ^{99}Mo.

b) For $^{99\text{m}}$Tc to be useful for nuclear medical applications, the activity $\mathcal{A}_2$ must be larger than 5 Ci at the time of the medical exam. Estimate with the help of the curve in which time interval the source can be used.

c) Why don't we see much activity from ^{99}Tc in the first 300 hours?

Problem 4 Bonus: How constant is the decay constant?

In most of our work in nuclear physics, we regard the decay constant λ as a true constant for a given nuclear species. However, you have studied two processes in which the nuclear decay rate could be sensitive to the chemical state of the atom. Discuss these two processes and explain how the atomic state might influuence the nuclear decay rate.
For a discussion and some examples of cases in which this can occur, see the review by G.T. Emery, `https://doi.org/10.1146/annurev.ns.22.120172.001121`