Problemset 3 (V2019),Ex4

a) We find

$\psi = R(r)\, Y_{l,m_l}(\theta,\varphi)$

which is given by

- $R_{nl}(r) = N_{nl}\, r^l\, e^{-\nu r^2}\, L_n^{(l+\frac{1}{2})}(2\nu r^2)$

some normalization factor

"Laguerre Polinomials"

or equivalently

through Hermitian Polynomials

- Y_{l,m_l} -> spherical harmonics

$= \sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\; \underbrace{P_l^m(\cos\theta)}_{\text{Legendre polinomials}}\; e^{im\varphi}$

b) possible values for l are

$l = N, N-2, \ldots, \begin{cases}1 \\ \text{or} \\ 0\end{cases}$, so for $N=5 \rightarrow l = \{1,3,5\}$

$N=6 \rightarrow l = \{0,2,4,6\}$

c)

N	$E_N\ (\hbar\omega)$	(n_r, l)	$\sum_{(n)} \overset{\text{spin T and L}}{2(2l+1)}$	total
0	3/2	1s	2	2
1	5/2	2p	6	8
2	7/2	2s, 1d	12	20
3	9/2	2p, 1f ~~3s, 2d, 1g~~	20	40
4	11/2	3s, 2d, 1g	30	70

c)

N	$\langle L^2\rangle_N$
0	0
1	$\frac{1\cdot 4}{2} = 2$
2	$\frac{2\cdot 5}{2} = 5$
3	$\frac{3\cdot 6}{2} = 9$

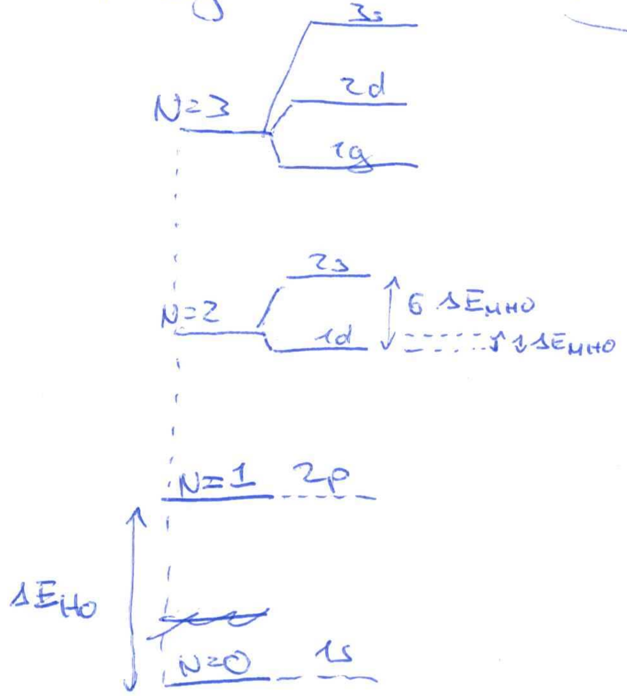
$\Rightarrow$

N	$L^2 \to l(l+1)$	$L^2 - \langle L^2\rangle_N$
0	$1s \to 0$	$0-0=0$
1	$2p \to 1(1+1) = 2$	$2-2=0$
2	$2s \to 0$	$0-5=-5$
	$1d \to 2(2+1) = 6$	$6-5=1$
3	$3s \to 0$	$0-9=-9$
	$2d \to 6$	$6-9=-3$
	$1g \to 3(2+1) = 12$	$12-9=3$

$\Rightarrow$ splitting of 2s and 1d !

— " — ditto

$\Rightarrow a_1$ must be negative in order to have higher l values at lower (more bound) energies



(however, note that the usual convention is to chose

$H_1 = \underbrace{-\mu}_{a_1}(L^2 - \langle L^2\rangle)\hbar\omega$, where

so $\mu > 0$

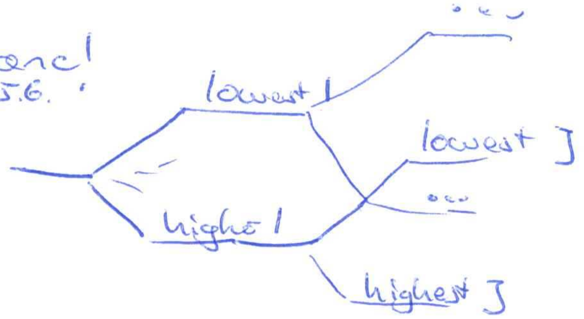
the relation of the splittings of the HO and ~~MHO~~ addition (L^2 term) is given by the ratio of the coefficients

~ "how large is μ" ?!

Ex 5

a)

~> see Krane! Fig 5.6. important!



[2] ——— 1s ——— [2] 2

deg. in HO — spectroscopic notation — deg. in this level — total number of nucleons til here [protons or neutrons]

• note: degeneracy here for protons or neutrons!
  = 2× for both of them

b) Magic numbers: 8, 20, 28, 50, 82, 126

↳ all of them predicted ☺

c) ^{27}Al odd proton, hole in $(1d_{5/2})^{-1}$ → $I^\pi = 5/2^+$; ~~same for~~

^{28}Si → $I^\pi = 0^+$

^{29}Si 1 neutron in $2s_{1/2}$ → $1/2^+$

^{31}P 1 p in $2s_{1/2}$ → $1/2^+$

^{32}S → 0^+

^{33}S → 1 n in $d_{3/2}$ → $3/2^+$

^{141}Pr → 1 p in $2d_{5/2}$ → $5/2^+$

same for e.g. $^{45}_{22}Ti_{23}$ → (2p, 3n) beyond Z = 20, N = 20, but we need to care only about the unpaired neutron

~~^{29}Al → look only at the unpaired neutron $(1d_{5/2})^3$~~

→ in $1f_{7/2}$ ⇒ $I^\pi_{gs} = 7/2^-$

d) $^{23}_{11}Na$ – expected spin (naively)

care only about ~~last~~ unpaired ~~neutron~~ proton,

$\rightarrow 1d_{5/2} \rightarrow I^{\pi} = \frac{5}{2}^{+}$

<u>however</u>: $I^{\pi}_{exp} = \frac{3}{2}^{(+1?)}$ — check up online

incomplete subshell, residual forces (beyond extremely indep. single part. shell modell)

$^{203}_{81}Tl$ – again, one would just regard the hole in $h_{11/2} \rightarrow I^{\pi} = 11/2^{-}$

<u>however</u>: $I^{\pi}_{exp} = \frac{1}{2}^{+} \Rightarrow$ due to pairing;

pairing energy <u>increases with ℓ</u>, such that it is more favourable to have this config. [last three shells given]

(actual) $(2d_{3/2})^4 (3s_{1/2})^2 (1h_{11/2})^{11}$ — particle number in that shell

then <u>even though</u> h has a higher energy the s.

(extr. ind. single part) $(2d_{3/2})^4 (3s_{1/2})^1 (1h_{11/2})^{12}$

$\Rightarrow$ • ~~extremely~~ independent single particle shell modell is just one extreme and cannot explain all effects. We need to take into account eg.

!

• deformation • <u>Many-particle</u> Shell model

!

• ~~[illegible]~~

in more realistic models.

However, it works incredibly well for a large number of nuclei, including ~~almost~~ most ~~all~~ :)

even-even, even-uneven nuclei in

$A < 150$ & $190 < A < 220$

[$\rightsquigarrow$ otherwise often large deformation]

$V_{HO+LS} = \frac{M\omega_0^2}{2} r^2 - K\hbar\omega_0 [2\vec{l}\cdot\vec{s} + \mu'(\ell^2 - \langle \ell^2 \rangle_N)]$

e) all terms

$= (n_r - 1) + \ell$ (or $n_r' + \ell$)

$$\Rightarrow E_{N\ell j} = \hbar\omega_0 \left[N + \frac{3}{2} + K \left\{ \begin{matrix} \ell \\ -\ell-1 \end{matrix} \right\} - \mu' \left(\ell(\ell+1) - \frac{N(N+3)}{2} \right) \right. \text{ for } \left\{ \begin{matrix} j = \ell + \frac{1}{2} \\ j = \ell - \frac{1}{2} \end{matrix} \right.$$