FYS3500 - Solutions to problem set 4

Spring term 2019

Problem 1 – in class: Shell model

a) The ground-state spin of ^{17}F is $J^\pi = 5/2^+$, and of the first excited state it is $J^\pi = 1/2^+$. The second excited state is $J^\pi = 1/2^-$. Give the configurations for protons and neutron of in the ground-state and first excited state and the second excited state.

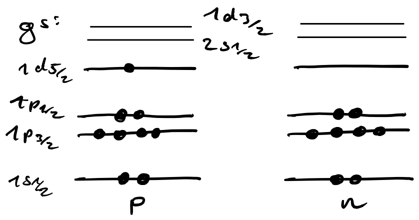
Problem 1

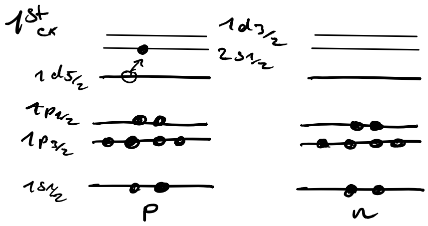
$^{17}_{9}F$, $J^\pi = \frac{5}{2}^+$ (g.s.) , $J^\pi = \frac{1}{2}^-$ (2nd ex)

$J^\pi = \frac{1}{2}^+$ (1st ex)

$\# p = 9$

$\# n = 8$

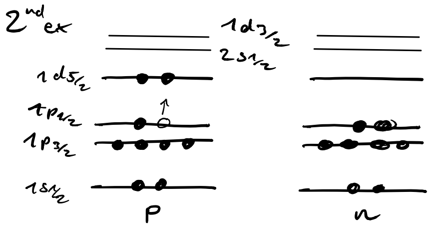




$\rightsquigarrow s_{1/2} \to J = \frac{1}{2}$

$\pi = (-1)^{l=0}$

$= + \checkmark$



$p_{1/2} \rightsquigarrow J = \frac{1}{2}$

$\pi = (-1)^{l=1} = -1$

$\checkmark$

Problem 2 – in class: Particle physics intro

a) *What are elementary particles?*
Point particles, that do not have an internal structure (so they are not composed of other particles) and don't have excited states.

b) *What are the force carriers of the different processes?*
Force carriers (which are also called *gauge bosons*); they have spin 1

- Weak interaction: $W^{\pm}$ and Z bosons.
- Strong interaction: gluons g
- electromagnetic interaction: photons γ

c) *Argue for the existence of antiparticles using the Dirac picture of the vacuum. Name at least one experimental evidence of antiparticles.*
The relativistic energy relation $E^2 = p^2c^2 + m^2c^4$ leads to the Klein-Gordon equation that also has negative energy solutions. Same yields for the Dirac equation. This can be explained using the Dirac picture of vacuum, where there are positive and negative energy states. For fermions, the negative energy states however are almost always filled (they cannot occupy the same states). The positive energy states are unoccupied. This is indistinguishable from the "normal" vacuum, since the momentum sums to 0, and the energy of the vacuum is 0 by definition (...). If one now removes an electron with negative energy $E = -E_p < 0$, momentum $-\boldsymbol{p}$, spin $-\boldsymbol{S}$ and charge $-e$ from the *vacuum* one remains with a state with a *hole* that has positive energy. This cannot be distinguished by adding a particle with positive energy $E = E_p > 0$, momentum $\boldsymbol{p}$, spin $\boldsymbol{S}$ and charge $+e$. Thus, Dirac can predicted a spin $1/2$ particle with the same mass as the electron, but opposite charge. This particle is called the *positron* and is referred to the as the *antiparticle* of the electron.

The positron was subsequently observed in a cloud chamber. The track observed in the chamber was equivalent to that of an electron, however in the magnetic field the track was bending in the opposite direction as an electron would (due to the opposite charge). But curvature of the track alone is not enough, it could still be a slow proton. This was discarded however, due to the different stopping powers (i.e. track lengths) of protons and electrons/positrons.

Problem 3

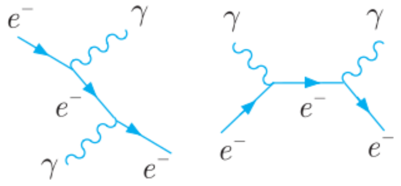
Draw the topologically distinct Feynman diagrams that contribute to the following process in lowest order

a) $\gamma + e^- \to \gamma + e^-$,

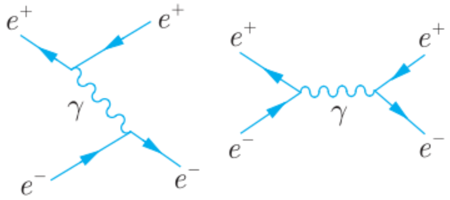
b) $e^+ + e^- \to e^+ + e^-$,

c) $\nu_e \bar{\nu}_e$ elastic scattering. (Hint: There are (at least) two such diagrams for each reaction)

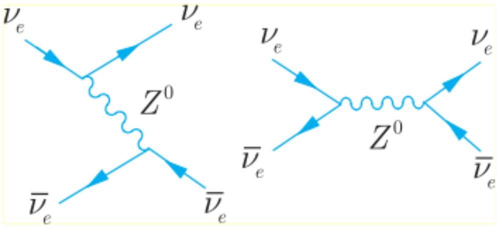
The topologically distinct diagrams for reactions (a) are given below.



Those for (b) are



and those for (c) are shown below.



Note on b) – for the second diagram also with Z^0 possible

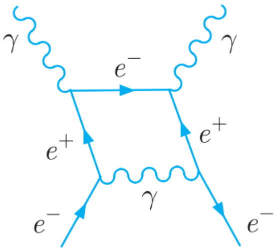
Problem 4

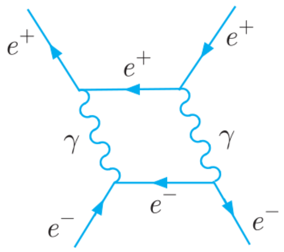
Draw one forth-order diagram for each of the reactions

a) $\gamma + e^- \to \gamma + e^-$,

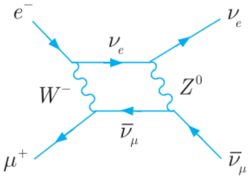
b) $e^+ + e^- \to e^+ + e^-$

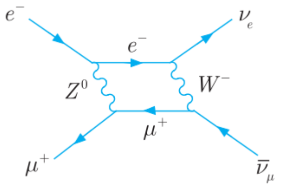
Two possible diagrams are shown in below. There are others.





c) and the two forth-order diagrams for $e^- + \mu^+ \to \nu_e + \bar{\nu}_\mu$.





Problem 5

For total centre-of-mass energies up to a fre GeV, the cross-section for the reaction $\nu_\mu + e^+ \to \mu + \nu_\mu$ is given by $\sigma = G_F^2 E^2/\pi$ in natural units, where G_F is the Fermi coupling constant. What is the cross-section in (pico)barns at an energy of E=3 GeV?

> *Substituting the values for E and* G_F*, gives* $\sigma = 3.9 \times 10^{-10}\text{GeV}^{-}2$*, and using the conversion factor 1* GeV^{-2} *= 0.389 mb, (taken eg. from the inside of the backcover of MS), to convert from natural to SI units, we obtain* $\sigma = 1.5 \times 10^{-10}\text{mb} = 0.15\text{pb}$

Problem 6

Which of the following reactions are allowed, and which are forbidden, by the conservation laws appropriate to weak interactions?

a) $\nu_\mu + p \to \mu^+ + n$,

b) $\nu_e + p \to e^- + \pi^+ + p$,

c) $\nu_\tau + e^- \to \tau^- + \nu_e$

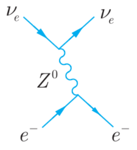
d) $\tau^+ \to \mu^+ + \bar{\nu}_\mu + \nu_\tau$.

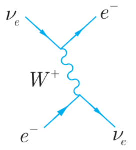
Weak interactions conserve baryon number B, charge Q and leptopn numbers L_e, L_μ and L_τ. They need not conserve the quantum numbers S, C or $\tilde{B}$. Of the decays given (a) violates L_μ conservation and (d) violates both L_μ and L_τ conservation. They are therefore both forbidden. The reactions (b) and (c) satisfy all the conservation laws and are allowed.

Problem 7

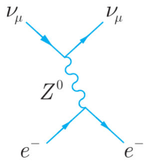
In MS, sec. 2.3.1. it is stated that electron neutrinos interact with electrons in a different wat from muon and tauon neutrinos. Justify this remark by considering the lowest-order Feynman diagrams for $\nu_e + e^- \to \nu_e + e^-$ and $\nu_\mu + e^- \to \nu_\mu + e^-$.

The electron neutrino may interact with electrons via both Z^0 and W^- exchange:





whereas, because of lepton number conservation, the muon neutrino can only interact via Z^0 exchange:



Note: A topological different diagram with W^- exists, but are not shown.

Problem 8

Which of the following reactions are allowed, and which are forbidden? If they are allowed, classify intro strong, electromagnetic and weak reactions.

a) $\Lambda \to \pi^+ + e^- + \bar{\nu}_e$
forbidden – violates baryon number conservation,

b) $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$
Allowed, involves neutrinos and is therefore a weak interaction,

c) $p + \bar{p} \to \pi^+ + \pi^- + \pi^0$
Conserves all quantum numbers; strong interaction,

d) $\Lambda + p \to K^- + 2p$
Conserves all quantum numbers; strong interaction,

e) $K^+ \to \pi^0 + \mu^+ + \bar{\nu}_\mu$
forbidden – violates muon lepton number conservation,

f) $K^+ \to \pi + +e^- + \bar{\nu}_e$
forbidden – violates charge conservation,

g) $K^- \to \pi - +e^+ + e^-$
weak interaction with Z^0,

h) $\gamma + p \rightarrow \pi^- + n$
Not allowed, charge conservation (if it was a π^+, it would be electromagnetic interaction, because it involves a photon)

i) $D^- \rightarrow K^+ + 2\pi^-$
Allowed interaction, but as it violates both strangeness and charm it must be a weak interaction.

j) $\pi - +p \rightarrow n + e^- + e^+$
Allowed, electromagnetic interaction.