FYS3500 - Solutions to Problem set 5

Spring term 2019

Problem 1 Dark matter – From last years exam

Show that, if there is a process $\chi\chi \to \chi\phi$ by which two cold (very low velocity) dark matter particles χ of mass m_χ annihilate to produce a single χ plus an additional massless dark particle ϕ, the remaining χ has 25% more energy than one of the cold χ's that collided.

From conservation of energy we find $2m_\chi = E^f_\chi + E_\phi$ and $E^i_\chi = m_\chi$. Conservation of momentum yields $p^f_\chi = p_\phi$. We can then use the invariant mass $E^{f,2}_\chi = m^2_\chi + p^{f,2}_\chi$ (and $E_\phi = p_\phi$):

$$2m_\chi = E^f_\chi + E_\phi = E^f_\chi + p_\phi = E^f_\chi + p^f_\chi \tag{1}$$

$$E^{f,2}_\chi = (2m_\chi - p^f_\chi)^2 = 4m^2_\chi + p^{f,2}_\chi - 4m_\chi p^f_\chi, \quad \text{but also (s.a.)} \quad E^{f,2}_\chi = m^2_\chi + p^{f,2}_\chi \tag{2}$$

$$m^2_\chi = 4m^2_\chi - 4m_\chi p^f_\chi \tag{3}$$

$$p^f_\chi = 3/4m_\chi \tag{4}$$

$$\to E^{f,2}_\chi = m^2_\chi + (3/4m_\chi)^2 \tag{5}$$

$$\to \frac{E^f_\chi}{m_\chi} = \frac{E^f_\chi}{E^i_\chi} = \frac{5}{4} \tag{6}$$

Theoughout, we have used the superscripts i and f to describe initial and final state.

Problem 2

Problem 3.4 in M&S
Note that it should be Table 3.4, not 3.5

The quantum numbers are:

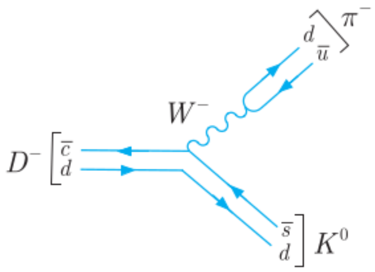
$$X^0: \ B=1,\ S=-1,\ C=0,\ \widetilde{B}=0\ ;\ Y^-: \ B=1,\ S=-2,\ C=0,\ \widetilde{B}=0\,.$$

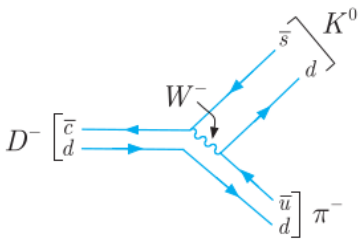
From their charges, and the definitions of $\widetilde{B}$, S, C and B, it follows that $X^0 = uds$ and $Y^- = dss$. The decay $Y^- \to \Lambda + \pi^-$ violates strangeness conservation and is a weak interaction, so we expect $\tau = 10^{-6} - 10^{-13}$ s. (The Y^- is in fact the so-called $\Xi^-(1321)$ state with a lifetime 1.6×10^{-10}s)

Problem 3

Problem 3.9 in M&S

(a) The quark compositions are: $D^- = d\bar{c}$; $K^0 = d\bar{s}$; $\pi^- = d\bar{u}$ and since the dominant decay of a c-quark is $c \to s$, we can have either of the diagrams shown below.





(b) The quark compositions are: $\Lambda = sud$; $p = uud$ and since the dominant decay of an s-quark is $s \to u$, we have the diagram: