FYS3500 - Solutions set 6

Spring term 2019

Problem 1

Discuss why the process $e^- + e^+ \to \gamma$ is not possible.
Use energy and momentum conservation to show that you get inconsistent results.

Problem 2

Describe briefly why it was necessary to introduce the strangeness and lepton numbers.
See M&S section on Kaons for strangeness (it's clearly stated in the text). For the lepton numbers, there could be different explanations: One could, for example use the absence of reactions like $\bar{\nu} + n \to p + e^-$, but existence of $\bar{\nu} + n \to p + e^+$ (called β-decay).

Problem 3 Space-time symmetries 1

a) Show that parity invariance leads to parity conservation, that is $[H, \hat{P}] = 0$, where

$$\hat{P}\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, t) = P_1 P_2 \cdots \Psi(-\boldsymbol{r}_1, -\boldsymbol{r}_2, \ldots, t) \tag{1}$$

b) *Why is the reaction $\pi^- + d \to n + n + \pi^0$ effectively forbidden for a π^- at rest, but proceeds at a normal rate for a strong reaction at higher energies?*
Hint: Think about what spins the 2 neutrons would need at low energies to obey total angular momentum conservation.
In the initial state, the spin of the π is 0 and the spin (J) of the deuteron is 1 (recall this from the nuclear physics lectures...); and for a pion at rest the orbital angular momentum is 0. So the total angular momentum is $J_i = 1$. The Q value of the reaction is only 0.5 MeV, so the final state will be in an s state ($L = 0$). To preserve total angular momentum in the reaction, the two neutrons must therefore be in a triplet spin state, which is forbidden by the Pauli principle. However, at higher energies the final state need not have zero orbital angular momentum, so that the two neutrons need not be in an s state ($L = 0$), and the reactions is allowed.

Problem 4 Space-time symmetries 2

a) *A neutral spin-2 meson M^0 can decay via the strong interaction to a $\pi^+\pi^-$ final state. Use this to deduce its parity and charge conjugation quantum numbers.*
See discussion on canvas! **Remember that spin-2 meson means $J = 2$, not necessarily $S = 0$.**

Problem 5 Relativistic kinematics

a) Problem A.1

For pions at rest, energy conservation gives $E_\mu + E_{\nu_\mu} = m_\pi^2 c^2$, and hence

$$m_\pi^2 c^4 = E_\mu^2 + E_\nu^2 + 2E_\mu (m_\pi^2 c^2 - E_\mu),$$

Using $E^2 = p^2 c^2 + m^2 c^4$ gives

$$m_\pi^2 c^2 = -m_\mu^2 c^2 + m_\nu^2 c^2 + 2E_\mu m_\pi^2 + (p_\nu^2 - p_\mu^2)$$

But momentum conservation is $\mathbf{p}_\mu + \mathbf{p}_\nu = \mathbf{0}$; i.e. $\mathbf{p}_\mu^2 = \mathbf{p}_\nu^2$. Thus,

$$E_\mu = \frac{(m_\pi^2 + m_\mu^2 - m_\nu^2)c^2}{2m_\pi}.$$

(This result has been used to obtain an upper limit on the mass of the muon neutrino from a measurement of E_μ.)

b) Problem A.4 in M&S

Use momentum conservation to find the momentum of X^0. Then test the two hypotheses by calculating the invariant masses of the initial and final states in each case using the masses given in Appendix E. The decay is $\Lambda \to p\pi^-$.