Problem 3

a), b) Decay probability determined through:

- "building" the α-particle ← Q-value needs to be positive
- knock on Coul-bar.
- Tunneling probability

Liquid drop: Coulomb term leads to α-decay of proton rich nuclei ?!

+ see problem 2

– neutron rich nuclei: generally prone to β^-, not to α-decay

c) • α-particle is mono-energetic, but different energies are possible due to decay in excited states

Higher L transfer less likely; parity needs to be taken into account

⇒ there are 'forbidden' decays

⊳ this explains simultaneously the deviation from the Q-value

Problem 4

a) $A_1(0) = A_2(0) = A_3(0) = 1{,}0\ \mu Ci$

$t_{1,1/2} = 1{,}0\ s, \quad \lambda_1 = \frac{\ln 2}{1{,}0\ s} = 0{,}693\ 1/s$

$t_{2,1/2} = 1{,}0\ h = 3600\ s, \lambda_2 = 1{,}925 \cdot 10^{-4}\ 1/s$

$t_{3,1/2} = 1{,}0\ d = 86400\ s, \lambda_3 = 8{,}023 \cdot 10^{-6}\ 1/s$

$A(t) = \lambda N(t)$

$N_1(0) = \frac{A_1(0)}{\lambda_1} = \frac{1{,}0 \cdot 10^{-6} \cdot 3{,}7 \cdot 10^{10}\ 1/s}{0{,}693\ 1/s} = 53380$

$N_2(0) = \frac{A_2(0)}{\lambda_2} = \frac{1{,}0 \cdot 10^{-6} \cdot 3{,}7 \cdot 10^{10}\ 1/s}{1{,}925 \cdot 10^{-4}\ 1/s} = 192207792$

$N_3(0) = \frac{A_3(0)}{\lambda_3} = \frac{1{,}0 \cdot 10^{-6} \cdot 3{,}7 \cdot 10^{10}\ 1/s}{8{,}023 \cdot 10^{-6}\ 1/s} = 4611741244$

b) $\Delta N_1 = N_1(0) - N_1(1\ s) = N_1(0)\left(1 - e^{-\lambda_1 1s}\right) = 26690$

$\Delta N_2 = N_2(0) - N_2(1\ s) = N_2(0)\left(1 - e^{-\lambda_2}\right) = 36996$

$\Delta N_3 = N_3(0) - N_3(1\ s) = N_3(0)\left(1 - e^{-\lambda_3}\right) = 37000$

c) $\Delta N_1 = N_1(0) - N_1(3600\ s) = N_1(0)\left(1 - e^{-3600\lambda_1}\right) \approx 53380$

$\Delta N_2 = N_2(0) - N_2(3600\ s) = N_2(0)/2 = 96103896$

$\Delta N_3 = N_3(0) - N_3(3600\ s) = N_3(0)\left(1 - e^{-3600\lambda_3}\right) = 131294792$

Problem 5

a) It is crucial that the halflife is short enough that we can measure some (sufficiently) many decays,
and long enough that we can after a time t_1 still measure decays.
With a halflife of $t_{1/2} \approx 5700\,y$ ^{14}C is a good candidate up to some 5-10 thousands of years. [uptake of objects of ^{14}C stops when they die, right!]

b) ~~radioactive tests~~ Test of nuclear weapons,
see eg. ctbto.org for more information

c) Apply $N = N_0 e^{-\lambda t}$ to both isotopes

$$\frac{99.28}{0.72} = \frac{e^{-\lambda_{238} t}}{e^{-\lambda_{235} t}} \Rightarrow t = 5{,}9 \cdot 10^9\,y$$

However, this is based on the (probably wrong) assumption of equal ~~masses~~ abundances in the start. Ask me for more detailed calcs. if you're interested.

Uranium isotopes, like all heavy ($A \gtrsim 56$) elements are produced in supernova explosions. The material ejected from this is used to build up new stars. A proper isotopic analysis of meteorits leads to the age of the solar system (not Big Bang!) of $4.55 \cdot 10^9\,y$.

d) After $2.5 \cdot 10^9\,y$, $(1 - e^{-\lambda t})$ of the nuclei will have decayed, so 32%.