FYS3500 - Solutions to problem set 9

Spring term 2019

Problem 1 – in class

a) Did you calculate all problems from the previous sets? Any questions?

b) Start with the problem below; after some examples of (a), focus especially (b to d).

Problem 2 γ-decay and Weisskopf units

The reduced transition probabilities are expressed in $e^2\text{fm}^{2L}$ for EL multipoles and in $\mu_N^2\text{fm}^{2L-12}$ for ML multipoles, see eg. Krane.

a) For the following transitions between levels, give all permitted γ-ray multipoles and indicate which multipole might be the most intense in the emitted radiation.
a) $\frac{9}{2}^- \to \frac{7}{2}^+$: E[**1**,3,5,7], M[2,4,6,8] b) $\frac{1}{2}^- \to \frac{7}{2}^-$: **M3, E4** c) $1^- \to 2^+$: **E1**, (E3, M2) d) $0^+ \to 0^+$: No gamma; internal conversion e) $3^+ \to 3^+$: M[**1**,3,5], E[**2**,4,6] e) $4^+ \to 2^+$: E[**2**,4,6], M[3,5]
f) $\frac{11}{2}^- \to \frac{3}{2}^+$: M[**4**,6], **E5**

b) A nucleus has the following sequence of states beginning with the ground state: $\frac{3}{2}^-, \frac{7}{2}^-, \frac{5}{2}^+, \frac{1}{2}^-$ and $\frac{3}{2}^-$ Draw a level scheme showing the intense γ transitions likely to be emitted and indicate their multipole assignment. Which of the transitions would you expect to be have the smallest chance to happen?

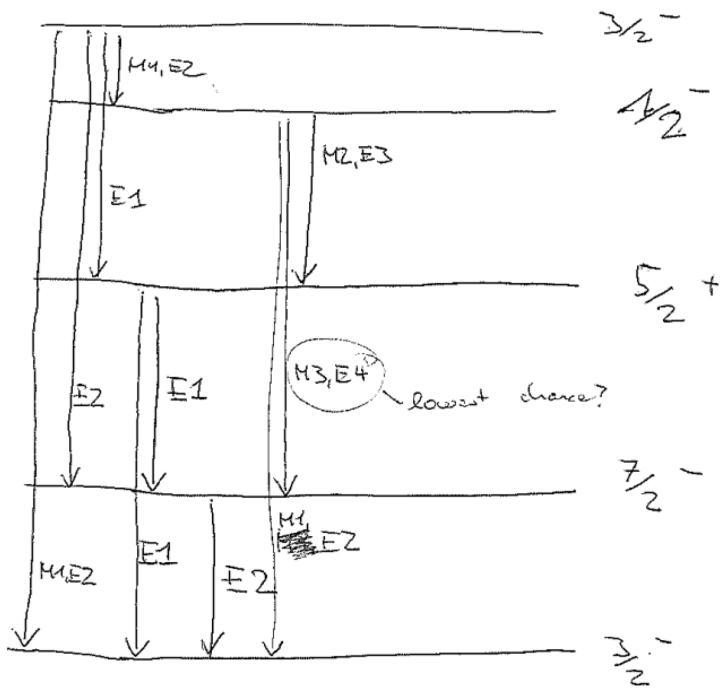


Figure 1

Answer: See discussion in class. If we to first order neglect the E_γ dependence – the energies are not given in the level scheme – The highest multipolarities & for the same multipolarity the magnetic transitions have the smallest chance to happen.

c) Which γ would you expect to be emitted in coincidence with each other (your timing resolution may be something like 50ns). How would a low-lying excited state with high spin behave?
Answer: Rule of the thumb: all transition cascades with E1, E2 & M1, maybe also M2. Rather less likely: The $3/2-, M1 \rightarrow 1/2-, M2 \rightarrow 5/2-, M3 \rightarrow 7/2-, M1 \rightarrow 3/2-$ transition.
The gs spins are often rather low, so one needs high multipolarities to decay down from a low-lying excited state with high spin. That increases their lifetime. They are called metastable, or isomeric states. **An important example of this is given below,** $^{99m}_{43}\text{Tc}_{56}$. – Look up the spin-parities to confirm this.

d) Assume now, that the daughter nucleus is populated by β-decay with a Q-value of 7 MeV from the $I = 3/2^-$ gs of a mother nucleus. Sketch the electron spectrum. Hint: Is this enough information? What else do you potentially need? If in class: Ask for the information. If at home: Make assumptions where necessary.
Answer: In class; Important is the superposition of the spectra, with the "correct" intensities of the different transitions and the "correct" end-point values. Potentially one would also see discrete lines of internal conversion.

e) For a light nucleus ($A = 10$), compute the ratio of the emission probabilities for electric quadrupole (E2) and magnetic dipole (M1) radiation according to the Weisskopf estimates. Consider all possible choices for the parities of the initial and final states.
Answer: Assuming a gamma-ray energy of 1 MeV, In the cases where both transitions are possible: 2.8×10^{-5}

f) Compare this to the ratio calculated for a heavy nucleus (A = 200).
Answer: Same assumptions; 1.5×10^{-3}

Problem 3 Medical isotope production

Technetium-99m ($^{99m}_{43}\text{Tc}_{56}$) is a metastable nuclear isomer of ^{99}Tc (itself an isotope of Tc) that is used in tens of millions of medical diagnostic procedures annually, making it the most commonly used medical radioisotope.[1]

The technetium ^{99m}Tc (excited state of ^{99}Tc) is a γ-emitter used in nuclear medicine to for example detect cervical tumors with a γ-camera.[2]

$$^{99m}\text{Tc} \rightarrow {}^{99}\text{Tc} + \gamma$$

^{99m}Tc is a byproduct of the β^--decay of ^{99}Mo:

$$^{99}\text{Mo} \rightarrow {}^{99m}\text{Tc} + e^- + \bar{\nu}_e$$

A simplified decay scheme is shown in Figure 2

[1] Tc-99m: `https://en.wikipedia.org/wiki/Technetium-99m`
[2] Based on Exercise 2.1 in `https://lpsc.in2p3.fr/schien/PHY113a/TD_radio_en_2011-2012.pdf`

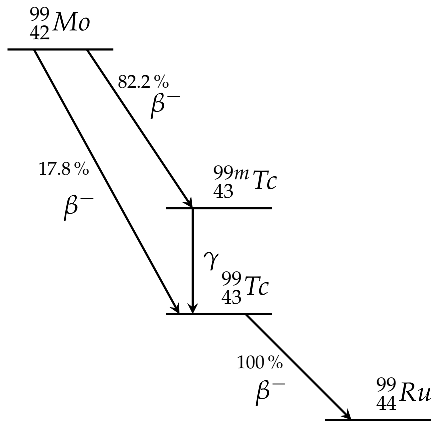


Figure 2: Simplified decay scheme of ^{99}Mo.

^{99}Mo can be produced for example in a cyclotron. Let's assume that the initial activity of a ^{99}Mo source is $A(0) = 8.5$ Ci and that source does not contain any Tc initially.

a) Plot the number of nuclei ($N_x(t)$ vs t) and activity ($\mathcal{A}_x(t)$ vs t) for ^{99}Mo, $^{99\text{m}}$Tc, ^{99}Tc and ^{99}Ru for the first 300 hours.
Hint: You may want to find informations such as half-lives etc. from `http://www.nndc.bnl.gov/chart/` or `https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`. Start by calculating the number of ^{99}Mo nuclei present at $t = 0$. You may either derive the equations for the radioactive decay series, or look them up. In any case, remember to take into account the branching in the decay of ^{99}Mo.
Answer: First we consider a decay chain $1 \to 2 \to 3$, where nuclei of type 3 are stable.

$$N_1(t) = N_0 e^{-\lambda_1 t} \qquad \text{(Parent)}$$

$$N_2(t) = N_0 \frac{\lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t})}{\lambda_2 - \lambda_1} \qquad \text{(Daughter)}$$

$$N_3(t) = N_0 \frac{\lambda_1 (1 - e^{-\lambda_2 t}) - \lambda_2 (1 - e^{-\lambda_1 t})}{\lambda_1 - \lambda_2} \qquad \text{(Stable)}$$

where $N_x(t)$ is the exponential law of radioactive decay for the different nucleis, N_0 is the number of nuclei present at $t = 0$ for the parent nucleus, λ_x is is the disintegration constant for the different nucleis and t is the time.
The related activities are (Equation 6.24 on page 169 and Equation 6.32 on page 171 in Krane)

$$\mathcal{A}_1(t) = \lambda_1 N_1(t)$$
$$\mathcal{A}_2(t) = \lambda_2 N_2(t)$$

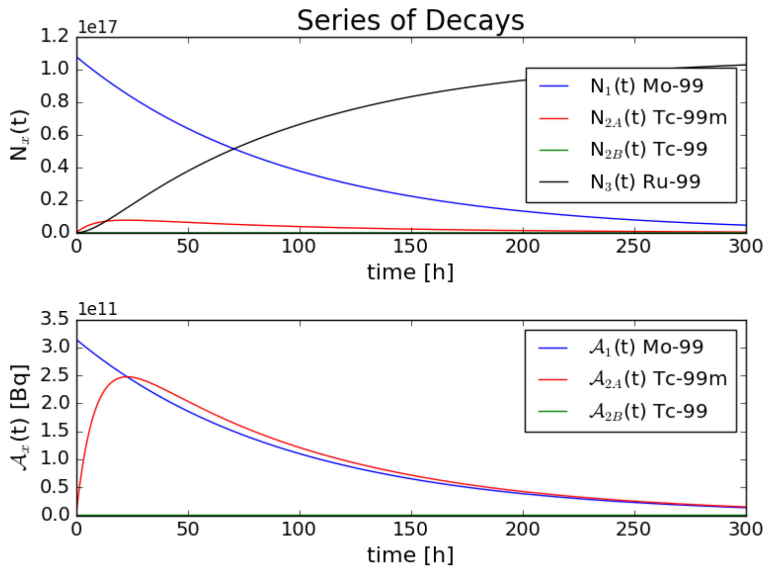
Since nuclei of type 3 are stable, the activity is zero.
For the radioactive decay of Mo you can use $N_1(t)$ and $N_3(t)$ as given above. However, ^{99}Mo decays to two isotopes with a certain probability, and therefore we have to take into account the weights w_{2A} and w_{2B} to decay into the daughter nuclei (^{99m}Tc and ^{99}Tc). The equations become

$$N_{2A}(t) = w_{2A} \cdot N_2(t) \qquad w_{2A} = \frac{\lambda_{2A}}{\lambda_{2A} + \lambda_{2B}} = \frac{\lambda_{2A}}{\lambda_t}$$

$$N_{2B}(t) = w_{2B} \cdot N_2(t) \qquad w_{2B} = \frac{\lambda_{2B}}{\lambda_{2A} + \lambda_{2B}} = \frac{\lambda_{2B}}{\lambda_t}$$

where $N_2(t)$ given above and $\lambda_2 = \lambda_t = \lambda_{2A} + \lambda_{2B}$. You can read about weighted decays on page 164 in Krane.

For the activity you can use $\mathcal{A}_1(t)$ and $\mathcal{A}_2(t)$ (one each for A and B) as in the simple case. There is no activity for ^{99}Ru because it is stable.



b) For $^{99\text{m}}$Tc to be useful for nuclear medical applications, the activity $\mathcal{A}_2$ must be larger than 5 Ci at the time of the medical exam. Estimate with the help of the curve in which time interval the source can be used.
Answer: 5 Ci are 1.85×10^{11} Bq. With the curve I estimate the activity of $^{99\text{m}}$Tc to be higher then this for the first 8-60 h.

c) Why don't we see much activity from ^{99}Tc in the first 300 hours?
Answer: $t_{1/2} \approx 2.5 \times 10^5$ y, so it decays only quite slowly in comparison to the other isotopes.

Problem 4 Bonus: How constant is the decay constant?

In most of our work in nuclear physics, we regard the decay constant λ as a true constant for a given nuclear species. However, you have studied two processes in which the nuclear decay rate could be sensitive to the chemical state of the atom. Discuss these two processes and explain how the atomic state might influuence the nuclear decay rate.
For a discussion and some examples of cases in which this can occur, see the review by G.T. Emery, `https://doi.org/10.1146/annurev.ns.22.120172.001121`