

# UNIVERSITY OF OSLO

## Faculty of Mathematics and Natural Sciences

**Exam in FYS3500 - Introduction to nuclear and particle physics**

**Day of exam: 6 June, 2019**

**Exam hours: 14:30-18:30**

**This examination paper consists of 8 page(s).**

**Appendices: None**

**Permitted materials: Chart of the nuclides, calculator**

*Make sure that your copy of this examination paper  
is complete before answering.*

100 points total

**Useful constants:**

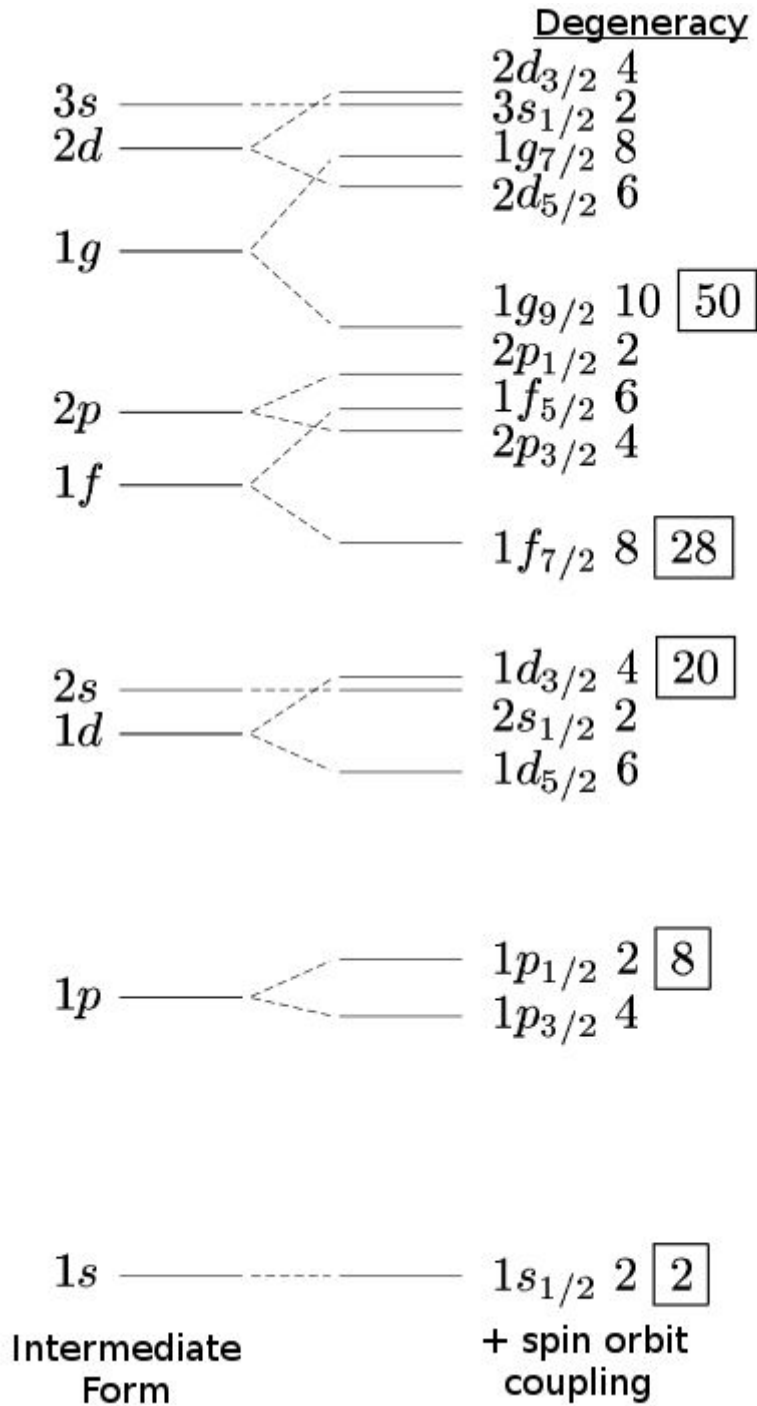
$$c = 3 \cdot 10^8 \text{ m/s}$$

$$\hbar c = 197 \text{ MeV-F}$$

$$G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$$

Particle	Quark composition	Mass (MeV/c <sup>2</sup> )
$\pi^+$	$u\bar{d}$	140
$\pi^0$	$u\bar{u}, d\bar{d}$	135
$K^+$	$u\bar{s}$	494
$K^0$	$d\bar{s}$	498
$n$	$udd$	940
$p$	$uud$	938
$\Lambda^0$	$uds$	1116

# Shell Model - Energy Levels



## 1. Concepts in particle physics (7 points)

Associate the following list of concepts with the symbols or formulae below:

- a) Flavor-changing neutral current
- b) Isospin symmetry
- c) Parity violation
- d) Color confinement
- e) Electroweak unification
- f) Lepton universality
- g) Lepton-quark symmetry

- 1)  $\Sigma^+$  (1189 MeV) =  $uus$ ,  $\Sigma^0$  (1193 MeV) =  $uds$ ,  $\Sigma^-$  (1197 MeV) =  $dds$
- 2)  $\Gamma(W^+ \rightarrow e^+ \nu_e) \approx \Gamma(W^+ \rightarrow \tau^+ \nu_\tau)$
- 3)  $e/(2\sqrt{2}\epsilon_0) = g_W \sin(\theta_W) = g_Z \cos(\theta_W)$
- 4)  $\Gamma_{\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau}(\cos\theta) \neq \Gamma_{\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau}(\cos(\pi - \theta))$
- 5)  $\Gamma(W^+ \rightarrow u \bar{d}') = 3 \Gamma(W^+ \rightarrow e^+ \nu_e)$
- 6)  $B_s^0(\bar{b}s) \rightarrow \mu^+ \mu^-$
- 7)  $I_3^c = Y^c = 0$

## 2. Multiple choice (3 points)

1. The half-life  $t_{1/2}$  for  $\alpha$ -decay for a chain of isotopes
  - a. increases with Q-value.
  - b. decreases with Q-value.
  - c. is independent of the Q-value.
2. The half-life  $t_{1/2}$  for  $\alpha$ -decay for a chain of isotopes
  - a. decreases with increasing deformation.
  - b. increases with increasing deformation.
  - c. is independent of deformation.
3. With increased excitation energy of the nucleus we find in general states with shorter half-life and
  - a. smaller width  $\Gamma$ .
  - b. larger width.
  - c. the same width.

### 3. Nuclear forces (4 points)

- a) The nuclear force is described as “charge independent”. Why don't we observe a bound system of two protons or two neutrons in nature?
- b) What is the typical range of the nuclear force and why is it so much shorter than for example the electromagnetic force?

### 4. Nuclear binding energy (16 points)

- a) Make a sketch of the binding energy per nucleon versus mass number  $A$ . Where is the maximum?
- b) Write down the binding energy formula and explain the different terms.
- c) From which term can we learn about the range of the nuclear force? Explain briefly.
- d) The stable light nuclei have roughly the same number of neutrons  $N$  and protons  $Z$ . For heavier nuclei there is a deviation from this  $N=Z$  line. In which direction is the deviation and which term is responsible? Explain briefly.
- e) Why are  $^{233}\text{U}$ ,  $^{235}\text{U}$  and  $^{239}\text{Pu}$  fissionable with high probability by thermal neutrons, while  $^{232}\text{Th}$  and  $^{238}\text{U}$  are not? Hint: Think about the  $\delta$ -term.
- f) The fission fragments of fissile nuclei are not distributed symmetrically in mass, why?

### 5. Shell model (6 points)

- a) Find and explain ground state spin and parity of  $^{43}\text{Sc}$ .
- a) Consider  $^{48}\text{Ca}$  ( $N=28$ ,  $Z=20$ ). If we now take one neutron from the level  $1f_{7/2}$  and promote it to the level  $2p_{3/2}$ , we would have an excited state. What are the possible values for the spin and parity of this excited configuration?

## 6. Gamma decay (10p)

- a. For decay from states in the nucleus with the initial state  $J_i^\pi$  to the final state  $J_f^\pi$ , what are the possible  $\gamma$  decays (E1, M1 etc). If more than one are possible which is the most probable?
  - i.  $1^+ \rightarrow 1^+$
  - ii.  $9/2^- \rightarrow 3/2^+$
  - iii.  $2^+ \rightarrow 2^-$
  - iv.  $0^+ \rightarrow 0^+$
- b. In an experiment you observe three  $\gamma$ -rays with following energies and multiplicities:
  - i. 100 keV, E2
  - ii. 200 keV, M1
  - iii. 300 keV, E1No higher energy gamma-rays were observed. *Construct the most probable level scheme(s), including spin assignment of the levels.*  
Hint: There are (at least) two possible solutions.
- c. As hinted upon, there are several possible solutions. Discuss *briefly* a possibility to distinguish between them (i.e. what additional measurement would be needed and why).

## 7. Radioactive decay (12 points)

A nucleus  $A$  is unstable and decays with the decay constant  $\lambda_A$  to a another unstable nucleus  $B$ . This decays further to the nucleus  $C$  with the decay constant  $\lambda_B$ . This is usually written as  $A \rightarrow B \rightarrow C$ .

- a) *Derive* the decay law, i.e. the number of particles of  $A$ ,  $N_A(t)$ , at a given time  $t$ , from the number of particles  $A$  at time zero,  $N_{A,0}$ .
- b) *Prove* that the number of particles  $B$  at time  $t$  is given by
$$N_B(t) = \frac{N_{A,0}\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}).$$
- c) Suppose there was a lump of  ${}^{76}_{32}\text{Ge}$  ( $\sim 1$  mol,  $6 \times 10^{23}$  atoms) just after the big bang ( $13.8 \times 10^9$  years ago). The half-life of  ${}^{76}_{32}\text{Ge}$ , which decays mainly by double-beta decay, is  $1.78 \times 10^{21}$  years. How many decays are there per week today?
- d) Why are the shapes of the energy spectra for  $\beta^+$  and  $\beta^-$  in nuclear decays different?

## 8. Medical application (4 points)

- a) What is the advantage of proton therapy versus conventional radiation therapy with electrons or X-rays?
- b) Sketch the proton radiation dose as a function of depth in the body.

### 9. Quark-gluon plasma (4 points)

Explain why the  $J/\Psi (c\bar{c})$  and  $Y(b\bar{b})$  states are expected to be suppressed in the quark-gluon plasma that is formed during high-energy collisions between heavy ions. Also, tell what happens to the bottom and charmed quarks when a  $J/\Psi$  or  $Y$  meson is “suppressed”, i.e., what sorts of charm and bottom particles emerge from the plasma.

### 10. Hadrons and quarks (8 points)

Consider the decay sequence  $\Lambda_b^0(udb) \rightarrow K^-(\bar{u}s) P_c^+$  followed by the decay  $P_c^+ \rightarrow p J/\Psi$ , where the  $P_c^+$  is an exotic hadron consisting of  $uudc\bar{c}$ .

- As you solve part (b), identify which interaction types are most likely to mediate the two decays (explain your answers).
- Draw the quark diagrams for the decay sequence.
- Describe qualitatively how to determine experimentally whether the  $P_c^+$  exists as a bound state in  $\Lambda_b^0$  decays.

### 11. The Higgs boson (8 points)

The decay rate of the Higgs boson  $H$  to fermion-antifermion pairs  $f\bar{f}$  is given by  $\Gamma(H \rightarrow f\bar{f}) = (N_c)\alpha_W(m_f^2/m_W^2)m_H$ .

- What is the factor  $N_c$  for quarks and leptons, i.e. what does it account for and how big is it?
- Predict the ratios of branching fractions (symbols only)  
 $B(H \rightarrow b\bar{b}) : B(H \rightarrow c\bar{c}) : B(H \rightarrow \tau^+\tau^-)$ .
- Describe how the Higgs boson can decay to pairs of massless photons ( $H \rightarrow \gamma\gamma$ ) or gluons ( $H \rightarrow gg$ ), i.e. suggest 1-2 likely Feynman diagrams for each.

### 12. The Standard Model (8 points)

- How many continuous variable parameters are needed to describe the entire Standard Model of fundamental particles and interactions? Explain your reasoning/accounting for the various contributions to the total.
- How many of these parameters are given by pure theory and how many need to be determined by experiment?

**13. Relativistic kinematics (10 points)**

A beam of negatively charged pions ( $\pi^-$ ) hits protons at rest. What is the minimum energy of the pion beam (or the minimum momentum if you prefer) needed to produce a  $\Lambda^0$ ? Hint: Describe what type of interaction this is most likely to be (let's say the cross-section is very large) and think about how to insure that the relevant quantum numbers are conserved. A correct symbolic answer will receive full credit, but it could be a good idea to plug in some numbers and check if your formula gives sensible results.