$$H = \sum_{ka} \hbar\omega_k a^+_{ka} a_{ka} + \frac{1}{2m}\underbrace{(p - eA)^2}$$

$$\rightarrow = \frac{p^2}{2m} - \frac{e}{m} A(r)\cdot p + \frac{e^2}{2m} A^2$$

$$p \rightarrow -i\hbar\nabla_r$$

$$p\cdot A\,\psi(r)$$

$$\rightarrow \nabla_r A(r)$$

$$\rightarrow = A\cdot p\,\psi(r)$$

$$H = \underbrace{\sum_{ka} \hbar\omega_k a^+_{ka} a_{ka}}_{\text{Felt}} + \underbrace{\frac{p^2}{2m} + V(r)}_{\text{Atom}}$$

$$\underbrace{-\frac{e}{m} A(r)\cdot p \left(+\frac{e^2}{2m} A^2\right)}_{\text{Vekselvirkning}}$$

$$H_{int} = -\frac{e}{m} A(r)\cdot p = H_{emis} + H_{abs}$$

$$H_{emis} = -\frac{e}{m}\sum_{ka} \sqrt{\frac{\hbar}{2V\epsilon_0\omega_k}}\, p\cdot\epsilon_{ka} a^+ e^{-i(kr-\omega t)}$$

$$H_{abs} = -\frac{e}{m}\sum_{ka} \sqrt{}\, p\cdot\epsilon_{ka} a\, e^{i(kr-\omega t)}$$

$$H = H_0 + H_{int}$$

$$U_0 = e^{-\frac{i}{\hbar} H_0 t} \qquad |\psi\rangle_I = U_0^\dagger |\psi\rangle_S$$

$$H_{int}^I = U_0^\dagger H_{int} U_0$$

$$i\hbar \frac{d}{dt}|\psi\rangle_I = H_{int}^I |\psi\rangle_I$$

$$|\psi(t)\rangle_I = U_{int} \cdot |\psi(0)\rangle_I$$

$$i\hbar \frac{d}{dt} U_{int} = H_{int}^I U_{int}$$

$$U_{int}(t_f, t_i) = \mathbb{1} - \frac{i}{\hbar} \int_{t_i}^{t_f} dt\, H_{int}^I(t) U_{int}(t, t_i)$$

$$= \mathbb{1} - \frac{i}{\hbar} \int_{t_i}^{t_f} dt\, H_{int}^I(t) + \left(-\frac{i}{\hbar}\right)^2 \int_{t_i}^{t_f} dt \int_{t_i}^{t} dt'\, H_{int}(t) H_{int}(t')$$

$$+ \left(-\frac{i}{\hbar}\right)^3 \int \int \int HHH + \cdots$$

Initial tilstand $|i\rangle$ ved t_i

Sluttilstand $|f\rangle$ t_f

$$H_0|i\rangle = E_i|i\rangle \Rightarrow U_0|i\rangle = e^{-\frac{i}{\hbar}E_i}|i\rangle$$

$$H_0|f\rangle = E_f|f\rangle \Rightarrow U_0|f\rangle = e^{-\frac{i}{\hbar}E_f}|f\rangle$$

$$\langle f|U_{int}^{I}(t_f,t_i)|i\rangle$$

$$= \langle f|i\rangle - \frac{i}{\hbar}\langle f|\int_{t_i}^{t_f} dt\, \underbrace{H_{int}^{I}(t)}_{U_0^{\dagger} H_{int} U_0}|i\rangle + \cdots$$

$$= \langle f|i\rangle - \frac{i}{\hbar}\langle f|H_{int}|i\rangle \underbrace{\int_{t_i}^{t_f} dt\, e^{\frac{i}{\hbar}(E_f - E_i)t}}_{\frac{1}{i\omega_{fi}} 2i \sin\frac{\omega_{fi}T}{2} e^{i\omega_{fi}\bar{t}}} + \cdots$$

$$\hbar\omega_{fi} = E_f - E_i$$

$$T = t_f - t_i \qquad \bar{t} = \frac{1}{2}(t_i + t_f)$$

Overgangssannsynlighet

$$W_{fi} = \left(\frac{2\sin\frac{1}{2}\omega_{fi}T}{\hbar\omega_{fi}}\right)^2 |\langle f|H_{int}|i\rangle|^2$$

Matrise element

$|i\rangle = |A, n_{k\alpha}\rangle \qquad |f\rangle = |B, n_{k\alpha} \pm 1\rangle$

Absorpsjon:

$$\langle B, n_{k\alpha}-1|H_{abs}|A, n_{k\alpha}\rangle$$

$$= -\frac{e}{m}\sqrt{\frac{\hbar}{2V\epsilon_0\omega_k}}\langle B|\vec{p}e^{i\vec{k}r}|A\rangle\cdot\vec{\epsilon}_{k\alpha}$$

$$\langle n_{k\alpha}-1|\underbrace{a_{k\alpha}|n_{k\alpha}\rangle}_{\sqrt{n_{k\alpha}}\,|n_{k\alpha}-1\rangle} e^{-i\omega_k t}$$

$$= -\frac{e}{m}\sqrt{\frac{\hbar n_{k\alpha}}{2V\epsilon_0\omega_k}}\langle B|pe^{ikr}|A\rangle\cdot\epsilon_{k\alpha}e^{-i\omega_k t}$$

Dipol approksimasjon $\qquad r_{atom} \simeq 10^{-10}m$

$k = \frac{2\pi}{\lambda}, \ \lambda \simeq 10^{-7}m$

$$k \cdot r = 2\pi \cdot \frac{r_{atom}}{\lambda} \simeq 10^{-2}$$

$$e^{ikr} \simeq 1 + \underbrace{ikr}_{10^{-2}} + \cdots \simeq 1$$

$$\langle B | p e^{ikr} | A \rangle \simeq \langle B | p | A \rangle = p_{BA}$$

$$p_{BA} = i m \omega_{BA} r_{BA}$$

$$\hbar \omega_{BA} = E_B - E_A \qquad r_{BA} = \langle B | \hat{r} | A \rangle$$

Emission

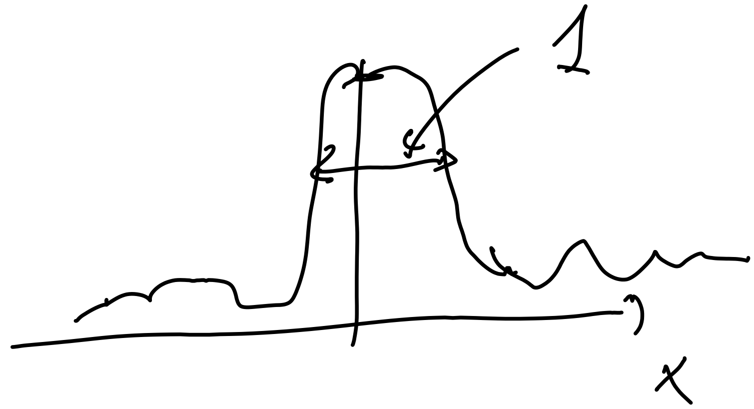
$$\langle B, n_{ka}+1 | H_{emis} | A, n_{ka} \rangle$$

$$= -\frac{e}{m} \sqrt{\frac{\hbar}{2V\epsilon_0 \omega_k}} \langle B | \hat{p} e^{-ikr} | A \rangle \cdot \vec{\epsilon}_{ka} \underbrace{\langle n_{ka}+1 | a_{ka}^{+} | n_{ka} \rangle}_{\sqrt{n_{ka}+1}} e^{i\omega_k t}$$

$$\left(\frac{2\sin\left(\frac{1}{2}\omega_{fi}T\right)}{\hbar\,\omega_{fi}}\right)^2 = \frac{T^2}{\hbar^2}\, f\left(\frac{\omega_{fi}T}{2}\right)$$

$$f(x) = \frac{\sin^2 x}{x^2}$$

$$f(0) = 1$$



$$f\left(\frac{\omega_{fi}T}{2}\right) \xrightarrow[T\to\infty]{} N\cdot\delta(\underbrace{\hbar\,\omega_{fi}}_{E_{fi}})$$

$$\int_{-\infty}^{\infty} f\Big(\underbrace{\frac{\omega_{fi}T}{2}}_{\frac{T}{2\hbar}E_{fi} = x}\Big)\, dE_{fi} = \int dE_{fi}\, N\,\delta(E_{fi}) = N$$

$$\underbrace{\qquad\qquad\qquad}_{\frac{2\hbar}{T}\cdot\underbrace{\int f(x)\,dx}_{\pi}}$$

$$N = \frac{2\pi\hbar}{T}$$

$$W_{fi} = \frac{2\pi}{\hbar} T \, |\langle f | H_{int} | i \rangle|^2 \, \delta(E_f - E_i)$$

Fermis gyldne regel

Overgangsrate:

$$w_{fi} = \frac{W_{fi}}{T} = \frac{2\pi}{\hbar} \underbrace{|\langle f | H_{int} | i \rangle|^2}_{} \, \delta(E_f - E_i)$$

<u>Abs:</u>

$$|\langle \, \rangle|^2 = \frac{e^2}{m^2} \frac{\hbar \, n_{ka}}{2V\epsilon_0 \omega_k} \, p_{BA} \cdot \epsilon_{ka}$$

<u>Em:</u>

$$|\langle \, \rangle|^2 = \frac{e^2}{m^2} \frac{\hbar (n_{ka} + 1)}{2V\epsilon_0 \omega_k} \, p_{BA} \cdot \epsilon_{ka}$$

Spontaneous emission.

$|i\rangle = |A, 0\rangle$ $\qquad$ $|f\rangle = |B, 1_{ka}\rangle$

$\sum_{ka}$

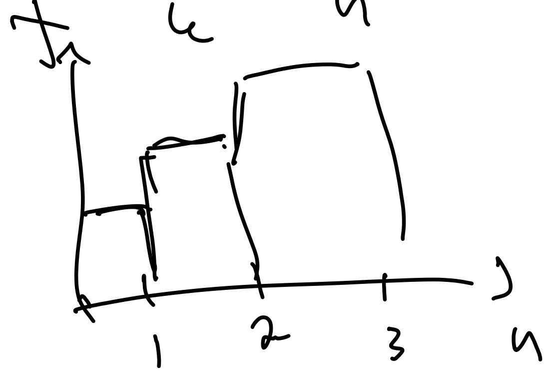
$$w_{BA} = \sum_{ka} \frac{2\pi}{\hbar} \left|\langle B, 1_{ka}| H_{emis} |A, 0\rangle\right|^2 \underline{\delta(E_f - E_i)}$$

$$\delta(E_B + \hbar\omega_k - E_A)$$

$$= \frac{V}{(2\pi)^3} \int d^3k \sum_a \frac{2\pi}{\hbar} \cdots$$

$$k_x = \frac{2\pi}{L} n_x$$

$$\int f(k)\, dk = \frac{2\pi}{L} \sum_n f(n) \qquad \sum_k = \sum_n =$$

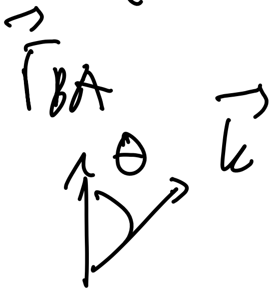


$$dk = \frac{2\pi}{L} \frac{dn}{1}$$

$$= \frac{V}{(2\pi)^3} \cdot \frac{2\pi}{\hbar} \int d\Omega \int dk\, \underset{\substack{\uparrow \\ \frac{\omega_k}{c}}}{k^2} \sum_a |\langle\ \rangle|^2 \delta(\ )$$

$$= \frac{e^2 \omega_{AB}^3}{8\pi^2 \epsilon_0 c^3 \hbar} \int d\Omega \sum_a |\epsilon_{ka} \cdot r_{BA}|^2$$

$$\sum_\lambda |\epsilon_{k\lambda} \cdot r_{BA}|^2 \overset{\text{oppg.}}{=} |r_{BA}|^2 - \frac{(\vec{r}_{BA} \cdot \vec{k})^2}{k^2}$$



$$\int d\Omega\,(1-\cos^2\theta) = \frac{8\pi}{3}$$

$$w_{BA} = \frac{e^2 \omega_{BA}^3}{3\pi\epsilon_0 \hbar c^3} |\underbrace{r_{BA}}_{\langle B|r|A\rangle}|^2$$