$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}\sum_i \gamma_i \left[L_i^\dagger L_i \rho + \rho L_i^\dagger L_i - 2 L_i \rho L_i^\dagger\right]$$

Example TLS at finite temperature, T

$$H = \frac{1}{2}\hbar\omega\sigma_z \qquad L_1 = \sigma_- \qquad L_2 = \sigma_+$$

$$\begin{pmatrix}0\\1\end{pmatrix} = |0\rangle \qquad \begin{pmatrix}1\\0\end{pmatrix} = |1\rangle$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \frac{\gamma_-}{2}\underbrace{\left[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_- - 2\sigma_-\rho\sigma_+\right]}_{\text{Emission}} - \frac{\gamma_+}{2}\underbrace{\left[\sigma_-\sigma_+\rho + \rho\sigma_-\sigma_+ - 2\sigma_+\rho\sigma_-\right]}_{\text{Absorption}}$$

$$\gamma_- \sim n+1 \qquad \gamma_+ \sim n \qquad \text{Eq: } n = \frac{1}{e^{\beta\hbar\omega}-1} \qquad \beta = \frac{1}{k_B T}$$

$$\frac{\gamma_+}{\gamma_-} = \frac{n}{n+1} = \frac{\frac{1}{e^{\beta\hbar\omega}-1}}{\frac{1}{e^{\beta\hbar\omega}-1}+1} = \frac{1}{1+e^{\beta\hbar\omega}-1} = e^{-\beta\hbar\omega}$$

$$\rho = \begin{pmatrix} p_1 & d \\ d^* & p_0 \end{pmatrix} \qquad p_1 + p_0 = 1$$

$$\underbrace{\frac{d\rho}{dt}} = -\frac{i}{\hbar}\underbrace{[H,\rho]} - \frac{\gamma_-}{2}\underbrace{\left[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_- - 2\sigma_-\rho\sigma_+\right]} - \frac{\gamma_+}{2}\underbrace{\left[\sigma_-\sigma_+\rho + \rho\sigma_-\sigma_+ - 2\sigma_+\rho\sigma_-\right]}$$

$$\frac{d}{dt}\begin{pmatrix} p_1 & d \\ d^* & p_0 \end{pmatrix} = -i\omega\begin{pmatrix} 0 & d \\ -d^* & 0 \end{pmatrix} - \frac{\gamma_-}{2}\begin{pmatrix} 2p_1 & d \\ d^* & -2p_1 \end{pmatrix} - \frac{\gamma_+}{2}\begin{pmatrix} 2p_0 & d \\ d^* & -2p_0 \end{pmatrix}$$

$$\dot{p}_1 = -\gamma_- p_1 + \gamma_+ \overset{1-p_1}{p_0} = -\gamma_- p_1 + \gamma_+ - \gamma_+ p_1 = -\gamma p_1 + \gamma_+$$

$$\dot{d} = \left(-i\omega - \frac{\gamma_-}{2} - \frac{\gamma_+}{2}\right)d \qquad \gamma_- + \gamma_+ = \gamma$$

$$= \left(-i\omega - \frac{\gamma}{2}\right)d$$

$$p_1(t) = \frac{\gamma_+}{\gamma} + \left(p_1(0) - \frac{\gamma_+}{\gamma}\right)e^{-\gamma t}$$

$$d(t) = d(0)\, e^{-(i\omega + \frac{\gamma}{2})t}$$

$t \to \infty$

$$P_1 = \frac{\gamma_+}{\gamma} \qquad P_0 = 1 - P_1 = 1 - \frac{\gamma_+}{\gamma} = \frac{\gamma - \gamma_+}{\gamma} = \frac{\gamma_-}{\gamma}$$

$d \to 0$

$$\frac{P_1}{P_0} = \frac{\gamma_+/\gamma}{\gamma_-/\gamma} = \frac{\gamma_+}{\gamma_-} = e^{-\beta\hbar\omega}$$

Bloch vector

$$x = 2\,\mathrm{Re}\,d = (x(0)\cos\omega t - y(0)\sin\omega t)\, e^{-\frac{\gamma}{2}t}$$

$$y = 2\,\mathrm{Im}\,d = (x(0)\sin\omega t + y(0)\cos\omega t)\, e^{-\frac{\gamma}{2}t}$$

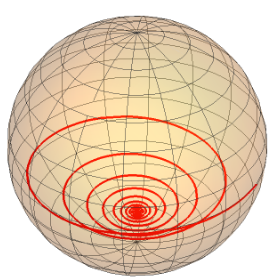
$$z = P_1 - P_0 = \frac{\gamma_+ - \gamma_-}{\gamma} + \left[z(0) - \frac{\gamma_+ - \gamma_-}{\gamma}\right] e^{-\gamma t}$$

$T = 0: \quad \frac{\gamma_+}{\gamma_-} = 0 \qquad \gamma_+ = 0 \qquad \gamma = \gamma_-$

$t \to \infty \qquad z \to -1$

$T = \infty \quad \frac{\gamma_+}{\gamma_-} = 1 \qquad \gamma_+ = \gamma_-$

$t \to \infty \qquad z \to 0$



$x(0) = 1$

$y(0) = z(0) = 0$

$\underbrace{\beta\hbar\omega}_{\frac{\hbar\omega}{k_B T}} = 1{,}6$