Solutions to problem set 11

11.1 Teleporting a unitary transformation

A teleports the state $|\psi\rangle$ to B, B performs the unitary operation U on the two qubits and teleports the resulting qubit back to A. Each teleportation requires one entangled pair and two bits of classical information.

The only worry is whether the entanglement between the two qbits that is created by the operation U will survive the teleportation. To convince oneself about this, one can use the Schmidt decomposition of the state. Let us call the state of the two qubits after U

$$|\chi\rangle_{14} = \sum_i d_i |\chi_i\rangle_1 \otimes |\phi_i\rangle_4$$

where we follow the convention in the lecture notes that the qubit to be teleported is number 1, the two qubits in the entangled pair is numbers 2 and 3, with 3 being the one to end in the teleported state. The qubit at B that is entangled with qubit 1 is number 4. At the end we want the entanglement to be transferred to number 3, which is at A. The teleportation protocol for one of the states $|\chi_i\rangle$ is (compare to Eqs. (3.11) and (3.16) in the lecture notes)

$$|\phi_i\rangle_{123} = |\chi_i\rangle_1 \otimes |\phi^-\rangle_{23} = \frac{1}{2}\sum_k |B_k\rangle_{12} \otimes V_k |\chi_i\rangle_3$$

where $|B_k\rangle_{12}$ are the Bell states defined in Eq (3.12) of the lecture notes. We then have

$$\begin{aligned}
|\chi\rangle_{14} \otimes |\phi^-\rangle_{23} &= \sum_i d_i |\phi_i\rangle_{123} \otimes |\phi_i\rangle_4 = \frac{1}{2}\sum_i d_i \sum_k |B_k\rangle_{12} \otimes V_k |\chi_i\rangle_3 \otimes |\phi_i\rangle_4 \\
&= \frac{1}{2}\sum_k |B_k\rangle_{12} \otimes V_k \sum_i d_i |\chi_i\rangle_3 \otimes |\phi_i\rangle_4 = \frac{1}{2}\sum_k |B_k\rangle_{12} \otimes V_k |\chi\rangle_{34}
\end{aligned}$$

We see that qubits 3 and 4 end in the desired entangled state if we apply the inverse of the V_k depending on the outcome of the measurement on qubits 1 and 2. Thus, the entanglement is transferred during teleportation.