Problem set 9

9.1 Teleporting a unitary transformation

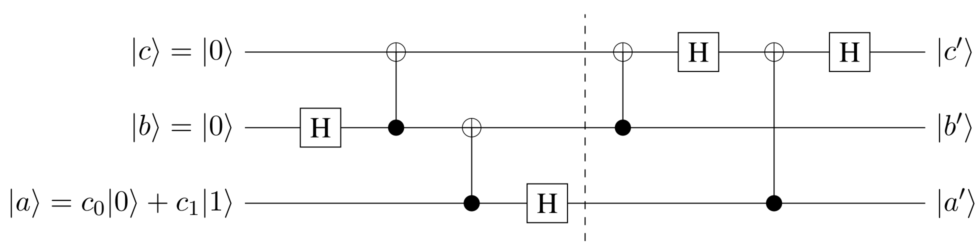
A has a qubit in the state $|\psi\rangle$ and B has a qubit in state $|\phi\rangle$. They also has a supply of shared entangled pairs, each in the state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$$

with A having the first qubit and B the second qubit of each pair. They want to perform a unitary transformation U on the two qubits, so that the final state is $U(|\psi\rangle \otimes |\phi\rangle)$. How can they achieve this? They are able to do any local unitary operation on any number of qubits, ant they are allowed to use any number of entangled pairs (how many do they need?) and communicate classically (how many bits of classical information must be exchanged?).

9.2 Quantum gates for teleportation

We consider the following quantum circuit (ignore the vertical dashed line for the moment)



a) Confirm that the final state is a product state despite the entanglement created in the middle of the circuit and that $|c'\rangle = |a\rangle$ so that the state of the lowest qubit is teleported to the upper qubit.

b) We now measure the a and b qubits at the vertical dashed line and perform the two remaining CNOT gates based on the outcomes of these measurements. That is, they are local at qubit c. Check that we still find $|c'\rangle = |a\rangle$ at the end.

9.3 Quantum cloning of orthogonal states

The no-cloning theorem tells us that it is not possible to make a copy of an arbitrary initial state. However, if we know that the states we have to copy are not general, but selected from a set of orthogonal states, we can find a way to copy them.

a) Given two orthgonal states $|\psi\rangle$ and $|\phi\rangle$ for a single qubit, design a quantum circuit with two input qubits with the following properties. If the first qubit is in the state to be copied, which is always either $|\psi\rangle$ or $|\phi\rangle$ and the second qubit is in a standard state $|0\rangle$, the output is $|\psi\rangle|\psi\rangle$ or $|\phi\rangle|\phi\rangle$

depending on whether $|\psi\rangle$ or $|\phi\rangle$ was input on the first qubit (the circuit is not general, it will depend on which states $|\psi\rangle$ and $|\phi\rangle$ are used). Assume that you can use as elementary gates in the circuit all single qubit gates and CNOT.

b) Assume for simplicity that the two orthogonal states are $|0\rangle$ and $|1\rangle$. What is the output of the circuit if the input is the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$?