FYS 4130 Statistical Mechanics extra problems

Magnet system

Consider a system of N distinguishable non-interacting spins in a magnetic field H. Each spin has a magnetic moment μ and can point either parallell or anti-parallel to the the magnetic field. The energy of each state is

$$E = \sum_{i=1}^{N} -n_i \mu H,\ n_i = \pm 1$$

This system is a canonical ensemble.

a) Calculate the partition function for this system

$$Z = (2 \cosh \beta \mu H)^N$$

b) Use the partition function to calculate the internal energy and entropy of the system.

c) Use the partition function to calculate the average magnetisation of the system.

$$M = \sum_{i=1}^{N} \mu n_i$$

And the magnetic susceptibility

$$\chi = \left(\frac{\partial M}{\partial H}\right)_{\beta,N}$$

Ideal Gas

Consider a box of volume V, with N molecules of gas at pressure P. This is a canonical ensemble. The energy of each molecule is

$$E = \frac{1}{2m}(P_x^2 + P_y^2 + P_z^2)$$

a) Calculate the single particle partition function. The sum over the energies can be approximated here by an integral over the energy $d\epsilon$. Here you must use the density of states.

$$D(\epsilon) = \frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$$

b) The partition function for N particles is now

$$Z = \frac{Z_1^N}{N!}$$

Use the Helmholtz free energy $F = -kT\ln Z$ to find the pressure and the equation of state for an ideal gas.

$$PV = NkT$$

Black body radiation

This system is a grand canonical ensemble, it does not have a fixed number of particles, in this case photons. The chemical potential for photons is $\mu = 0$. Consider a cavity of volume V with black body radiation.

a) Calculate the grand canonical partition function. Here it is easier to calculate $\ln \Xi$. The result is a sum over frequencies ω_i.

$$\ln \Xi = \sum_i \ln(\frac{1}{1 - e^{-\beta\hbar\omega_i}})$$

Approximate the sum as an integral over $d\omega$. Here you must use the density of states:

$$D(\omega) = \frac{V\omega^2}{\pi^2 c^3}$$

b) Use the grand canonical partition function to find the energy density of the black body radiation.