FYS 4130 Statistical Mechanics

Homework 1 Jan 20, 2009

1) Gibbs-Duhem equation for a rubber band

Consider a rubber band of length L held at tension f. For displacements between equilibrium states

$$dE = TdS + fdL + \mu dn$$

where μ is the chemical potential of a rubber band and n is the mass or mole number of the rubber band.

a) Derive the Gibbs-Duhem equation for a rubber band.

Suppose an equation of state for a rubber band is $E = \theta S^2 L/n^2$
where θ is a constant, L is the length of the rubber band.

b) Determine the chemical potential $\mu(T, L/n)$.

c) Show that the equation of state satisfies the Gibbs-Duhem equation.

Solution:
$0 = sdT + Ldf + nd\mu$
$\mu = -\frac{1}{2\theta}\frac{T^2}{(L/n)}$

2) Stability Conditions

An experimentalist claims to find that a gas obeys the following conditions:

1) $(\partial P/\partial V)_T < 0$

2) $(\partial P/\partial T)_V > 0$

3) $(\partial \mu/\partial V)_T < 0$

4) $(\partial T/\partial V)_S > 0$

a) Identify which of these inequalities is guaranteed by stability.

b) Identify which pair of inequalities are inconsistent with each other and demonstrate why they are inconsistent.

Solution: a) 1 is from stability conditions, b) 2) and 4) are inconsistent.