FYS 4130 Statistical Mechanics

Homework 10 March 24, 2009

1) Relativistic Fermi Gas

Consider a gas of spin 1/2 fermions, relativistic so that $\epsilon = pc$ and the density of states including the degeneracy is:

$$g(\epsilon)D(\epsilon) = \frac{V}{\pi^2\hbar^3c^3}\epsilon^2$$

a) Calculate the fermi energy.

b) Calculate the ground state energy, Which is the total energy of the gas at temperature $T = 0$.

c) Calculate the pressure at $T = 0$.

Solution:

a) $\epsilon_f = (3\rho\pi^2)^{1/3}\hbar c$

b) $E_0 = v\rho\frac{3}{4}(3\rho\pi^2)^{1/3}\hbar c$

c) $P_0 = \frac{\hbar c}{4}(3\pi^2)^{1/3}\rho^{4/3}$

2) Van der Waals gas

The critical point for the Van der Waals gas is the point in the $P\,V$ isotherm at T_c, the critical temperature, where

$$\left(\frac{\partial P}{\partial V}\right)_T = 0.$$

Find expressions for the critical volume, pressure and temperature, V_c, P_c, T_c in terms of a and b.

Show that $P_cV_c = \frac{3}{8}NkT_c$ at the critical point.

Extra problem: Relativistic Fermi Gas

Consider a gas of spin 1/2 fermions, nonrelativistic so that $\epsilon = \frac{p^2}{2m}$.

a) Calculate the fermi energy.

b) Calculate the total energy of the gas at temperature $T = 0$.

c) Calculate the pressure at $T = 0$.

Solution:

a) $\epsilon_f = \frac{\hbar^2}{2m}(3\rho\pi^2)^{2/3}$

b) $E_0 = \frac{3}{5}N\epsilon_f$

c) $P_0 = \frac{\hbar^2}{15m\pi^2}(3\pi^2\rho)^{5/3}$