FYS 4130 Statistical Mechanics

Homework 11 April 14, 2009

1) 1-D, 3 particle Ising model

The Hamiltonian for a chain of 3 Ising spins $\sigma_i = \pm 1$ is given by:

$$H = -J\sum_{i=1}^{2} \sigma_i \sigma_{i+1}$$

a) Calculate the partition function.

b) Calculate the internal energy and specific heat.

c) Find the $<\sigma_2>$ and the spin correlation function $<\sigma_1\sigma_3>$.

$Z = 4(1 + \cosh 2\beta J)$
$U = -2J \tanh \beta J$
$<\sigma_2> = 0$
$<\sigma_1\sigma_3> = \tanh^2 \beta J$

2) Ising model

A lattice has $N + 1$ Ising spins $\sigma_i = \pm 1$ in an external magnetic field B. Each spin in the lattice has a coupling to a single spin σ_0. The Hamiltonian is:

$$H = -B\sum_{i=}^{N} \sigma_i - J\sum_{i=1}^{N} \sigma_i \sigma_0$$

a) Calculate the partition function

b) Find $m = <\sigma_i>$ and the correlation $<\sigma_0\sigma_i>$.

c) Calculate the average energy E. What is E in the limit where $B \to 0$ and $J \to 0$?

$Z = e^{\beta B}(2\cosh(\beta B + \beta J))^N + e^{-\beta B}(2\cosh(\beta B - \beta J))^N$
$<E> = -B <\sigma_i> - J <\sigma_0\sigma_i>$