FYS 4130 Statistical Mechanics

Homework 12 April 14, 2009

2) 3 level Ising Model

Consider a 1 dimensional lattice with N spins and no external magnetic field. The coupling between spins has energy $\epsilon = -JS_iS_{i+1}$. The Hamiltonian is given by:

$$H = -J\sum_{i=1}^{N} S_iS_{i+1}$$

Where $J > 0$ and the spins can have 3 values $S_i = 1, 0, -1$.

a) Find the partition function $Z(T,N)$ in terms of temperature and the number of spins. Assume periodic boundary conditions. Use the transfer matrix method on page 150 in the notes. In this case it will be a 3×3 matrix.

b) What is the approximate partition function for large N?

c) Use this large N partition function to find the internal energy.

2) Mean Field for magnet

Consider a system of N atoms with spins $\sigma_i = \pm 1$. Each spin has an interaction with its nearest neighbors with coupling J each spin has z nearest neighbors.

$$H = -B\sum_{i=1}^{N} \sigma_i - J\sum_{i=1}^{N} \sigma_i\sigma_{i+1}$$

a) For the case $J = 0$, find the average magnetization $m = <\sum \sigma_i>$ and magnetic susceptibility $\chi = m/B$.

b) What are these values in the limit of weak field $B\beta \ll 1$?

c) Use the mean field approximation to find m and χ when $J > 0$.

d) At what temperature T_c is χ infinite?

$$T_c = zJ/K$$