

FYS 4130 Statistical Mechanics

Homework 13 April 28, 2009

1) Stochastic variables

A stochastic variable X can have values $x = 1, 3$. The variable Y can have values $y = 2, 4$. The joint probability density is:

$$P_{X,Y}(x, y) = \sum_{i=1,3} \sum_{j=2,4} p_{i,j} \delta(x - i) \delta(y - j)$$

- a) compute $\langle X \rangle$ and $\langle Y \rangle$ and the correlation $\langle XY \rangle$ for the case $p_{1,2} = p_{1,4} = p_{3,2} = p_{3,4} = 1/4$
- b) compute $\langle X \rangle$ and $\langle Y \rangle$ and the correlation $\langle XY \rangle$ for the case $p_{1,2} = p_{3,4} = 0$ and $p_{1,4} = p_{3,2} = 1/2$
- c) Are X and Y independent or correlated for these two cases?

2) Random Walk

Consider a random walk in one dimension with step length a and time interval τ . Each step has probability p to go forward and probability $q = 1 - p$ to go backward.

- a) For a large number of steps, find the gaussian probability distribution $P(x, t)$ which is the probability of finding the particle at a point between x and $x + dx$ at time t .
- b) Verify that this probability distribution satisfies the differential equation:

$$\frac{\partial n}{\partial t} + v \frac{\partial n}{\partial x} - D \frac{\partial^2 n}{\partial x^2} = 0$$

Find the constants v and D .