FYS 4130 Statistical Mechanics

Homework 14 May 5, 2009

1) Brownian Motion

Drops of oil of mass m are allowed to fall to the floor in still air at temperature T. Each drop is released at rest from the same point, at a height h above the floor.

Estimate the radius $r(h)$ of the patch of oil formed on the floor, using the following assumptions:

i) that the radius is determined by the Brownian motion of the falling drops

ii) that the time a drop takes to reach terminal velocity is much smaller than the time it takes to reach the floor. Assume it travels at the terminal velocity.

$r = \sqrt{\frac{4kTh}{mg}}$

2) Relaxation time for a 2 level system

Consider a system of N noninteracting atoms which have two energy levels $E = \pm\epsilon$. The system is brought into contact with a heat reservoir at temperature T. The system then evolves into thermal equilibrium with the heat reservoir. Each atom has probability per unit time $w_+(T)$ for transition from the lower energy level to the upper energy level. And probability per unit time $w_-(T)$ for transition from the upper energy level to the lower energy level.

a) Write down the master equations for $n_+(t)$ and $n_-(t)$. Where $n_+(t)$ is the number of atoms with energy $E = +\epsilon$ and $n_-(t)$ is the number of atoms with energy $E = -\epsilon$.

b) What is the value of w_+/w_- at equilibrium?

c) The nonequilibrium states can be characterized by $n(t) = n_-(t) - n_+(t)$. What is the master equation for $n(t)$?

d) What is n_{eq}, the equilibrium value of $n(t)$?

e) The system starts out in the state $n(0)$ and evolves into the state n_{eq}. What is the equation for $n(t)$? What is the relaxation time for the system to come to equilibrium ?

$\frac{w_+}{w_-} = e^{-2\beta\epsilon}$

$n_{eq} = N \tanh \beta\epsilon$

The relaxation time $\tau = \frac{1}{w_+ + w_-}$

1) Langevin Equation

The Langevin equation for a single particle with mass m moving in 2 dimensions is:

$$m\dot{v} = -\alpha v + F(t)$$

where

$$< F_i(t)F_j(0) >= 2\alpha kT\delta(t)\delta_{ij}$$

a) Find a solution for $v(t)$
Hint: multiply the equation by $e^{(\alpha t/m)}$ and calculate $< v_x(t)v_x(0) >$.

b)Show that the solution for $< v_x(t)v_x(0) >$ is consistent with the equipartition principle.