FYS 4130 Statistical Mechanics

Homework 2 Jan 27, 2009

1) Spin magnet system

Consider a system of N spin 1/2 particles with magnetic moment μ in an external magnetic field B so that each particle has either energy $+\mu B$ if the spin points opposite the B field or energy $-\mu B$ if the spin points along the B field.

a) Use Boltzmann's formula for entropy and the Stirling approximation to calculate the entropy of the system as a function of temperature.

b) Calculate the partition function for this system.

c) Calculate the entropy using the partition function.

d) Calculate the average magnetization $< m >$, and the magnetic susceptibility χ_T.

Solution:

$Z = (2\cosh\beta\mu B)^N$

$S/k = N\ln(2\cosh\beta\mu B) - \beta\mu B N\tanh\beta\mu B$

$< m >= N\mu\tanh(\beta\mu B)$

$\chi_T = N\beta\mu^2\frac{1}{\cosh^2\beta\mu B}$

3) Two level gas

A lattice gas consists of a lattice of N sites, each of which can be either empty, in which case the energy is 0, or occupied by one particle in which case the energy is ϵ.

a) Find the partition function for this system. Compare this to the partition function for the spin magnet system.

b) Calculate the average energy.

c) Calculate the entropy and temperature of the system.

Solution:

$Z = (1 + e^{-\beta\epsilon})^N$

$E = \frac{N\epsilon}{(e^{\beta\epsilon}+1)}$

$S/k = N \ln(1 + e^{-\beta\epsilon}) - \beta N\epsilon \frac{e^{-\beta\epsilon}}{1+e^{-\beta\epsilon}}$

2) Negative Temperature

a) Sketch the behavior of the entropy and temperature as a function of energy.

b) When the system is in a state with negative temperature and is brought into thermal contact with a reservoir at constant positive temperature, which way will the heat flow?