FYS 4130 Statistical Mechanics

Homework 5 Feb 17, 2009

1) Partition functions

Consider a system of noninteracting, identical and distinguishable particles.

a) Use the canonical partition function to calculate the energy $U(T, V, N)$ and the entropy $S(T, V, N)$.

b) Use the grand canonical partition function to calculate the energy $U(T, V, N)$ and calculate N as a function of the chemical potential μ.

Solution:

Both methods should give the same energy:

$$U = NkT^2 \left(\frac{\partial Z_1}{\partial T}\right)_V \frac{1}{Z_1} \tag{1}$$

$$N = \frac{e^{\beta\mu} Z_1}{1 - e^{\beta\mu} Z_1} \tag{2}$$

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2. Ideal Gas

Consider a box containing an ideal gas at pressure P and temperature T

a) Use the grand canonical partition function to find the equation of state. Write fugacity $z = e^{\beta\mu}$ as a function of the pressure and temperature.

b) The walls of the box have N_0 absorbing sites and each site can absorb one particle. The energy of an absorbed particle is $-\epsilon$.

Calculate the grand canonical partition function for the absorbed particles. And the average number of absorbed particles.

Solution:

$$P = z\left(\frac{2\pi m}{h^2}\right)^{3/2}(kT)^{5/2} \tag{3}$$

$$< N >= \frac{N_0}{1 + z^{-1}e^{-\beta\epsilon}} \tag{4}$$

3. Gibbs entropy formula

Show that the microcanonical entropy formula $S = klnW$ is consistent with the Gibbs entropy formula. $S = -k\sum_i P_i ln P_i$.