FYS 4130 Statistical Mechanics

Homework 6 Feb 24, 2009

1) Black body radiation

Consider an atom in a cavity containing black body radiation. When the atom is in the state a with energy E_a, there is a probability per unit time that it will make a transition to a state b with energy $E_b < E_a$, emitting a photon of frequency $\omega = (E_a - E_b)/\hbar$ in a state k. This probability per unit time is given by $P[(a, n_k) \rightarrow (b, n_k + 1)]$ when there are already n_k photons in the state k. The probability for the reverse process (absorption) is given by $P[(b, n_k) \rightarrow (a, n_k - 1)]$. From quantum electrodynamics one calculate approximately:

$$P[(a, n_k) \rightarrow (b, n_k + 1)] = (n_k + 1)P[(a, 0) \rightarrow (b, 1)]$$

$$P[(b, n_k) \rightarrow (a, n_k - 1)] = n_k P[(b, 1) \rightarrow (a, 0)]$$

a) The principle of detailed balance states that the rate of transition from $s \rightarrow s'$ is equal to the rate of the reverse transition from $s' \rightarrow s$, ie:

$$P(a)P[(a, n_k) \rightarrow (b, n_k + 1)] = P(b)P[(b, n_k) \rightarrow (a, n_k - 1)]$$

Where $P(a)$ is the probability for the atom to be in state a. Find the occupation number n_k as a function of temperature.

b) Use the density of states for photons

$$D(\omega) = \frac{V\omega^2}{\pi^2 c^3}$$

to calculate the Planck distribution for the energy spectrum of the radiation $u(w)d\omega$.

Solution:

a) $n_k = \frac{1}{e^{\beta\hbar\omega} - 1}$

b) $U(\omega) = \frac{V}{\pi^2 c^3} \frac{1}{e^{\beta\hbar\omega} - 1} \hbar\omega^3$

1) Neutrino background of the universe.

In the early universe, the matter and photons were in thermal equilibrium. As the temperature fell with expansion to below $T \sim 10^{11} K$, the neutrinos decoupled from the other particles. In this regime the neutrinos can be treated as noninteracting, massless, relativistic particles.

a) Calculate the number density of the neutrino background of the universe.

b) Calculate the entropy density for the neutrino background.

$N/V = \frac{8\pi}{h^3c^3}(kT)^3 I_2$

$S/V = \frac{8\pi}{h^3c^3} k^4 T^3 I_3$