FYS 4130 Statistical Mechanics

Homework 7 March 3, 2009

1) Neutrino background for massive neutrinos

Previously, to calculate the properties of the neutrino background of the universe, we assumed the neutrinos were noninteracting, relativistic, massless particles. The mass of the neutrino is small, but nonzero. Consider the neutrino background as a gas of noninteracting, relativistic, particles with mass.

a) Calculate the density of states for the massive neutrinos. Use the relativistic relation for momentum and energy: $E^2 = P^2c^2 + m^2c^4$.

b) Assuming that the mass of the neutrino is small, keep only the first order terms in the mass of the neutrino. Calculate the number density and energy density of the neutrino background.

c) Assume the neutrino mass is $m_\nu \sim 10^{-3}eV$, at what temperature does the correction for the mass of the neutrino become important?

Solution:

$$D(E) = \frac{8\pi V}{h^3c^2}(E^2/c^2 - m^2c^2)^{1/2}E$$

The correction from the mass is 10% at $T = \sqrt{5/7}\frac{mc^2}{\pi k}$
For $mc^2 = 10^{-3}eV$, $T = 3.12K$.

2) 3 Dimensional Debye Solid

In the Debye model for a 3D solid of N atoms, the energy per mode is given by $E = \hbar\omega$. There are $3N$ allowed modes with the highest mode at the Debye frequency ω_D Which is defined by

$$\int_0^{\omega_D} d\omega D(\omega) = 3N$$

a) Find an expression for the energy and heat capacity.

b) What is the temperature dependence of C_v in the limit T $\rightarrow \infty$?

c) What is the temperature dependence of C_v in the limit T $\rightarrow 0$?

Solution:

For $T \rightarrow \infty$, $U = 3NkT$, $C_v = 3Nk$

For $T \rightarrow 0$, $U = \frac{9N}{(\hbar\omega_D)^3}\frac{\pi^4}{15}4k^4T^3$