FYS 4130 Statistical Mechanics

Homework 8 March 10, 2009

1) Entropy of the Universe

The entropy of the universe has contributions from photons, neutrinos and ordinary matter. The matter in the universe is predominantly in the form of baryons. Assume the visible universe has a radius of $r = 10^10$ years and that the mass fraction of baryons is $\Omega_B = 4\%$. The expansion rate is $H = 70km/s/Mps$

a) Estimate the number of baryons in the visible universe. The number of photons is estimated to be $N_\gamma = 10^{88}$.

b) Calculate the entropy of the baryons using the relation for entropy and number density. Compare the entropy of baryons to the entropy of the photons.

Solution:

a) $N_B \approx 10^{81}$

b) $S_\gamma/S_B \approx 10^7$

2) Entropy of the Earth
The temperature of the earth is stable because the heat coming in from radiation from the sun is balanced by heat radiated from the earth as black body radiation. The entropy is not balanced. Calculate the entropy of the incoming radiation and the entropy of the outgoing radiation.

$G = 6.6726 \times 10^{-11} m^3 kg^{-1} s^{-2}$
$k = 1.3806 \times 10^{-23} JK^-1$
mass of the proton $m_P = 1.673 \times 10^{-27} kg$
1 parsec $1pc = 3.08 \times 10^{16} m$
1 year $= 3.1558 \times 10^7$ s.
Temperature of the sun $T_S = 5800K$
Temperature of the earth $T_E = 290K$
Radius of the earth $R_E = 6375m$

Entropy flux from radiation on earth
$\Delta S = 670J/ks$

Extra problem 1: Zero Point energy

Consider a 3 dimensional isotropic solid of N atoms treated as harmonic oscillators, with the Debye density of states. In addition to the energy in the excitations of the oscillators (the phonons) each oscillator has a zero point energy $\epsilon_0 = \hbar\omega/2$. Find the zero point energy of the system.

$D(\omega) = \frac{3V\omega^2}{2\pi^2 v^3}$

$E_0 = \frac{9}{8}N\hbar\omega_D$