FYS 4130 Statistical Mechanics

Homework 18 Feb 10, 2009

1) Zipper

A zipper has N links. Each link has two states: in state 1 the link is closed and has energy 0, in state 2 the link is open with energy ϵ. The zipper can only unzip from the left end and a link at position s cannot open unless all the links to the left (1,2,...,s-1) are already open.

a) Find the partition function for the zipper.

b) In the low temperature limit $\epsilon \gg kT$, find the mean number of open links.

$Z = \frac{1-e^{\beta\epsilon(N+1)}}{1-e^{-\beta\epsilon}}$

$\overline{s} \approx e^{-\beta\epsilon}$ for $\epsilon \gg kT$

2) Chemical potential of a gas

Consider a classical gas of interacting particles. The interaction potential between particles is $u(r)$

a) Using the canonical partition function, show that

$$\mu = \mu_0 + \Delta\mu$$

Where μ_0 is the chemical potential for an ideal gas.

$$\mu_0 = kT \ln \frac{N\Lambda^3}{V}$$

and

$$\Delta\mu = -kT \frac{\partial}{\partial N} \left(\ln \frac{Q_N}{V^N} \right)$$

where Q_N is the configuration integral

$$Q_N = \int d^{3N} r e^{-\beta u(r)}$$

b) In the mean field approximation, assume the particles cannot enter inside a hard repulsive core of radius $r = d$, and that the particles move in an average potential $\overline{u}$ when $r > d$.

Show that

$$\overline{u} = \frac{\rho}{2} \int_d^\infty dr 4\pi r^2 u(r) = -\rho a$$

c) Show that

$$\Delta\mu = \Delta\mu_1 + \Delta\mu_2 = \overline{u} - kT \ln(1 - \rho b)$$

here, b is defined by the reduced volume $V' = (1 - \rho b)V$ available to the particles out side the hard cores of the other particles.