FYS 4130 Statistical Mechanics

Homework 1 Jan 21, 2010

1) Thermodynamics of a rubber band

Consider a rubber band of length L held at tension f. The energy of the rubber band is given by

$$U = TS + fL + \mu n$$

where μ is the chemical potential of a rubber band and n is the mass or mole number of the rubber band. For displacements between states the first law relates diferentials of the extensive variables.

$$dU = TdS + fdL + \mu dn$$

a) Derive the Gibbs-Duhem equation for a rubber band. The Gibbs-Duhem equation relates the the diferentials of the intensive variables, dT, df, $d\mu$.

Suppose an equation of state for a rubber band is $E = \theta S^2 L/n^2$
where θ is a constant, L is the length of the rubber band.

b) Determine the chemical potential $\mu(T, L/n)$.

c) Show that the equation of state satisfies the Gibbs-Duhem equation.

Solution:
$0 = sdT + Ldf + nd\mu$
$\mu = -\frac{1}{2\theta}\frac{T^2}{(L/n)}$

2) Stability Conditions

An experimentalist has measured a gas and claims to find that the gas obeys the following conditions:

1) $(\partial P/\partial V)_T < 0$

2) $(\partial P/\partial T)_V > 0$

3) $(\partial \mu/\partial V)_T < 0$

4) $(\partial T/\partial V)_S > 0$

For an ideal gas the first law is:

$$dU = TdS - PdV + \mu dN$$

And the Gibbs-Duhem equation is:

$$0 = SdT - VdP + Nd\mu$$

a) Which of these inequalities is a condition for thermodynamic stability?

b) Identify which pair of inequalities are inconsistent with each other and demonstrate why they are inconsistent. You can use Maxwell relations or the Gibbs-Duhem equation to find relations between the partial derivatives.

Solution: a) 1 is from stability conditions, b) 2) and 4) are inconsistent.