FYS 4130 Statistical Mechanics

Homework 12 may 3, 2010

1) Mean Field for magnet

Consider a system of N atoms with spins $\sigma_i = \pm 1$. Each spin has an interaction with its nearest neighbors with coupling J each spin has z nearest neighbors.

$H = -B\sum_{i=1}^{N}\sigma_i - J\sum_{i=1}^{N}\sigma_i\sigma_{i+1}$

a) For the case $J = 0$, find the average magnetization $m = \langle\sum\sigma_i\rangle$ and magnetic susceptibility $\chi = m/B$.

b) What are these values in the limit of weak field $B\beta \ll 1$?

c) Use the mean field approximation to find m and χ when $J > 0$.

d) At what temperature T_c does χ diverge?

$T_c = zJ/K$

2) Random Walk

Consider a random walk in one dimension with step length a and time interval τ. Each step has probability p to go forward and probability $q = 1-p$ to go backward.

a) For a large number of steps, find an expression for $P_N(s)$, the probability of landing at position s after N number of steps. Where $s = R - L$ is the position along the x axis, R is the number of steps to the right and L is the number of steps to the left.

b) Find the gaussian probability distribution $P(x,t)$ which is the probability of finding the particle at a point between x and $x+dx$ at time t.F(Use stirling approximation)

c) Verify that this probability distribution satisfies the differential equation:

$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} - D\frac{\partial^2 n}{\partial x^2} = 0$

Find the constants v and D.