

## FYS 4130 Statistical Mechanics

### Homework 13 May 10, 2010

#### 1) Boson Gas (exam problem from 2009)

A gas of free bosons in three dimensions is in thermal equilibrium with temperature  $T$ . The energy of a particle with momentum  $p$  has the unusual form  $E = A|p|^{1/2}$  where  $A$  is a constant.

- a) Calculate the chemical potential of the gas in the classical limit as a function of density and temperature.
- b) What is the internal energy per particle in the gas?
- c) Quantum statistics must be used when the temperature is reduced. Show that the gas will then have a transition to a condensed phase and find the corresponding critical temperature  $T_c$  as a function of the density.
- d) What fraction of the total number of particles in the gas has condensed when the temperature is  $T = T_c/2$ ?

In this problem you can make use of the following value of the Riemann zeta function:

$$\zeta_R(6) = \frac{\pi^6}{945}$$