FYS 4130 Statistical Mechanics

Homework 3 Feb 5, 2010

1) 2 Dimensional Ideal Gas

Consider an ideal gas in 2 dimensions.

a) Calculate the 1 particle partition function.

b) Calculate the free energy and the equation of state.

c) You can also calulate the 1 particle partition function by changing the sum over energy states to an integral over continuous momentum states. Calculate the density of states and use this to find the 1 particle partition function.

Solution:

a) $Z_1 = \frac{A}{\Lambda^2}$, $A =$ area, $\Lambda = (\frac{2\pi\beta\hbar^2}{m})^{1/2}$

b) $P = NkT/A$

c) $D(\epsilon) = \frac{m}{2\pi\hbar^2}$

2) Boson Gas

Consider a gas of spin zero particles of mass m and chemical potential μ. Starting with the Bose distribution and the density of states:

$$D(\epsilon) = \frac{2\pi}{\hbar^3}(2m)^{3/2}\epsilon^{1/2}$$

a) Calculate the internal energy U. Leave the answer in terms of a dimensionless integral.

b) What happens to the occupation numbers at low temperature?

Solution:

a) $U = \frac{2\pi}{\hbar^3}\frac{(2m)^{3/2}}{\beta^{5/2}}\int_0^\infty dx \frac{x^{3/2}}{e^{-\mu\beta}e^x - 1}$

3) Probability distributions

Consider the Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein statistics.

a) What are the differences between them in terms of distiguishable particles and occupation numbers.

b) Why is the distinction between the Fermi-Dirac, Bose-Einstein and Maxwell-Boltzmann distributions unimportant at high temperature? How would you characterize high temperature in this case?

The thermal wavelength is smaller than the average distance between particles for the gas to be treated classically

$$\rho\Lambda^3 \ll 1$$

$$T \gg \frac{\rho^{3/2}h^2}{2\pi mk}$$