FYS 4130 Statistical Mechanics

Homework 5 Feb 26, 2010

1. Ideal Gas in a box

Consider a box containing an ideal gas at pressure P and temperature T

a) Use the grand canonical partition function to find the equation of state. Find $e^{\beta\mu}$ as a function of the pressure and temperature.

b) The walls of the box have N_0 absorbing sites and each site can absorb one particle. Assume the number of particles in the gas is much larger than the number of absorbing sites. The energy of an absorbed particle is $-\epsilon$.

Calculate the grand canonical partition function for the absorbed particles. And the average number of absorbed particles. You can treat each absorbing site as a separate system in equilibrium with the gas. First find $< N_s >$, the average number of particles in a single site.

Solution:

$$P = e^{\beta\mu}\left(\frac{2\pi m}{h^2}\right)^{3/2}(kT)^{5/2} \tag{1}$$

$$< N >= \frac{N_0}{1 + e^{-\beta\mu}e^{-\beta\epsilon}} \tag{2}$$

2) Ultra-relativistic gas

In a relativistic gas you can ignore the mass and the energy is then $E = pc$. For a gas with n particles in a 3D box of volume V

a) Calculate the volume in phase space.

b) Find the entropy and temperature, pressure and equation of state.

c) Use the grand canonical partition function to find the chemical potential for the gas.

3) Canonical and grand canonical ensembles

Consider a system of noninteracting, identical and distinguishable particles.

a) Use the canonical partition function to calculate the energy $U(T, V, N)$ and the entropy $S(T, V, N)$ in terms of the single particle partition function Z_1.

b) Use the grand canonical partition function to calculate the energy $U(T, V, N)$ and calculate N as a function of the chemical potential μ.

Solution:

Both methods should give the same energy:

$$U = NkT^2 \left(\frac{\partial Z_1}{\partial T} \right)_V \frac{1}{Z_1} \tag{3}$$

$$N = \frac{e^{\beta\mu} Z_1}{1 - e^{\beta\mu} Z_1} \tag{4}$$

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