FYS 4130 Statistical Mechanics

Homework 6 Mar 5, 2010

1) Chemical potential of a gas, exam problem from 2008

Consider a classical gas of interacting particles. The interaction potential between particles is $u(r)$

a) Using the canonical partition function, show that the chemical potential of the interacting gas can be written

$$\mu = \mu_0 + \Delta\mu$$

Where μ_0 is the chemical potential for a noninteracting ideal gas.

$$\mu_0 = kT \ln \tfrac{N\Lambda^3}{V}$$

and

$$\Delta\mu = -kT \tfrac{\partial}{\partial N} \left(\ln \tfrac{Q_N}{V^N} \right)$$

where Q_N is the configuration integral, (see page 68 in the notes)

$$Q_N = \int d^{3N} r e^{-\beta u(r)}$$

b) In the mean field approximation, assume the particles cannot enter inside a hard repulsive core of radius $r = d$, and that the particles move in an average potential $\overline{u}$ when $r > d$.

Show that

$$\overline{u} = \tfrac{\rho}{2} \int_d^\infty dr 4\pi r^2 u(r) = -\rho a$$

c) Show that

$$\Delta\mu = \Delta\mu_1 + \Delta\mu_2 = \overline{u} - kT \ln(1 - \rho b)$$

here, b is defined by the reduced volume $V' = (1 - \rho b)V$ available to the particles out side the hard cores of the other particles.

2) Van der Waals gas

The Van der Waals gas equation of state is given by

$$P = \frac{NkT}{V - Nb} - a\frac{N^2}{V^2}$$

The critical point for the Van der Waals gas is the point on the P V plot of the T_c isotherm, where

$$\left(\frac{\partial P}{\partial V}\right)_T = 0$$

T_c is the critical temperature.

a) Find expressions for the critical volume, pressure and temperature, V_c, P_c, T_c in terms of a and b.

b) Show that $P_c V_c = \frac{3}{8} N k T_c$ at the critical point.