FYS 4130 Statistical Mechanics

Homework 7 March 12, 2010

1) Gas of rigid spheres

A gas of rigid, hard core particles in a volume V may be described by dividing the volume into many small cells, each of volume a^3. There can be 0 or 1 particles in each of these cells, in both cases the energy is zero. Assume there are no other interactions between the particles. The total number of particles is not fixed and the system is in equilibrium with a reservoir of temperature T and chemical potential μ.

a) compute the mean number of particles in each cell and find the particle density ρ of the gas as a function of T and μ

$$\rho = \frac{1}{a^3} \frac{e^{\beta\mu}}{1 + e^{\beta\mu}}$$

b) Use the grand canonical ensemble to find the pressure $P(\rho, T)$ of the gas.

2) Black body radiation

Consider a cavity of volume V containing black body radiation. The energy of a photon is a function of the frequency $\epsilon = \hbar\omega$. This is grand canonical ensemble of photons which does not have a fixed number of particles. The chemical potential for photons is $\mu = 0$.

a) Calculate the grand canonical partition function. Here it is easier to calculate $\ln \Xi$. The result is a sum over frequencies ω_i

$$\ln \Xi = \sum_i \ln \left(\frac{1}{1 - e^{-\beta\hbar\omega_i}} \right)$$

Approximate the sum as an integral over $d\omega$. Here you must use the density of states for a massless, relativistic particle

$$D(\omega) = \frac{V\omega^2}{\pi^2 c^3}$$

b) Use the grand canonical partition function to find the energy density of the black body radiation.

c) Find the pressure and entropy density

d) Find the relation between pressure and energy density.

3) Two dimensional photon gas - previous exam problem

Consider a two dimensional photon gas with degeneracy g = 2 at temperature T. Using the formula for the mean occupation number of the photon energy state ϵ,

$$\langle n_\epsilon \rangle = \frac{1}{exp(\epsilon/kT) - 1}$$

where $\epsilon = pc = h\nu$, p is the momentum and ν is the frequency of the photon.

a) find the number of photons dn_ϵ per unit volume in the energy interval $(\epsilon, \epsilon + d\epsilon)$ and derive the two dimensional Planck formula for the spectral radiation energy density.

$$\rho(\nu, T) = A\frac{\nu^2}{e^{h\nu/kT} = 1}$$

where $\rho(\epsilon, T)d\epsilon = \epsilon dn_\epsilon$.

b) Consider the limiting cases: $h\nu \gg kT$, find two dimensional analogue of the Wien radiation law in this case.

c) Consider the case where d $h\nu \ll kT$. Find the two dimensional analogue of the Rayleigh-Jeans radiation law for this case.

d) Derive the Stefan-Boltzmann law for spatial energy density $u(T) = U/V$.

e) Find the heat capacity C_v, entropy and pressure of the photon gas.

e) Show that $PV = U/2$.