FYS 4130 Statistical Mechanics

Homework 9 April 8, 2010

1) Neutrino background of the universe.

In the early universe, the matter and photons were in thermal equilibrium. As the temperature fell with expansion to below $T \sim 10^{11}K$, the neutrinos decoupled from the other particles. In this regime the neutrinos can be treated as noninteracting, massless, relativistic particles.

a) Calculate the number density of the neutrino background of the universe.

b) Calculate the entropy density for the neutrino background.

$N/V = \frac{8\pi}{h^3c^3}(kT)^3 I_2$

$S/V = \frac{8\pi}{h^3c^3}k^4T^3 I_3$

2) Neutrino background for massive neutrinos

Previously, to calculate the properties of the neutrino background of the universe, we assumed the neutrinos were noninteracting, relativistic, massless particles. The mass of the neutrino is small, but nonzero. Consider the neutrino background as a gas of noninteracting, relativistic, particles with mass.

a) Calculate the density of states for the massive neutrinos. Use the relativistic relation for momentum and energy: $E^2 = P^2c^2 + m^2c^4$.

b) Assuming that the mass of the neutrino is small, keep only the first order terms in the mass of the neutrino. Calculate the number density and energy density of the neutrino background.

c) Assume the neutrino mass is $m_\nu \sim 10^{-3}eV$, at what temperature does the correction for the mass of the neutrino become important?

Solution:

$$D(E) = \frac{8\pi V}{h^3c^2}(E^2/c^2 - m^2c^2)^{1/2}E$$

The correction from the mass is 10% at $T = \sqrt{5/7}\frac{mc^2}{\pi k}$
For $mc^2 = 10^{-3}eV$, $T = 3.12K$.

3) Entropy of the Universe

The entropy of the universe has contributions from photons, neutrinos and ordinary matter. The matter in the universe is predominantly in the form of baryons. Assume the visible universe has a radius of $r = 10^10$ years and that the mass fraction of baryons is $\Omega_B = 4\%$. The expansion rate is $H = 70km/s/Mps$

a) Estimate the number of baryons in the visible universe. The number of photons is estimated to be $N_\gamma = 10^{88}$.

b) Calculate the entropy of the baryons using the relation for entropy and number density. Compare the entropy of baryons to the entropy of the photons.

Solution:

a) $N_B \approx 10^{81}$

b) $S_\gamma / S_B \approx 10^7$