Answers to problem set 3
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3.1

Zeptojoules or yoctojoules

3.2

a) For any reasonable units, $S, T, E < 10^{100} < \Omega(E)$.

b) $T \sim 1$, $E, S \sim N$, $\Omega(E) \sim e^N$

3.5

a) $A_N = \frac{1}{N!} \prod_i^N \left(A - (i-1)\pi(2r)^2\right)$

b) $b = 2\pi r^2$

c) $P(A - Nb) = Nk_B T$

3.7

a)

$$\Omega(E) = \frac{\left(\frac{3N}{2}\right)! \left(\frac{5N}{2}\right)!}{\left(\frac{8N}{2}+1\right)!} E^{\frac{8N}{2}+1}$$
$$E_1^{\max} = \frac{3}{8}E$$

b)

$$\rho(E_1) \propto \exp\left(-\frac{(E_1^{\max} - E_1)^2}{2\sigma^2}\right), \quad \sigma^2 \propto \frac{E^2}{N}$$
$$\langle E_1 \rangle = E_1^{\max}$$
$$\frac{\sqrt{\langle (E_1 - E_1^{\max})^2 \rangle}}{N} \propto \frac{1}{\sqrt{N}} \quad \text{(Remember that energies are extensive.)}$$

3.9

a)

$$P(n) = \binom{T}{n} \frac{1}{K^n} \left(1 - \frac{1}{K}\right)^{T-n}$$

b)

$$\begin{aligned} \langle 1 \rangle &= 1 \\ \langle n \rangle &= a \\ \langle n^2 \rangle &= a^2 - a \\ \sigma^2 = \langle (n - \langle n \rangle)^2 \rangle &= a \end{aligned}$$

c)

$$a = \frac{T}{K}$$

d)

$$\frac{1}{\sigma_K^2} = -\frac{1}{N_0} \frac{K}{K-1}$$

e) When you find your answer you can check its normalisation directly.

3.11

a) The three relations are

$$\begin{aligned} \left(\frac{\partial T}{\partial V}\right)_{S,N} &= -\left(\frac{\partial P}{\partial S}\right)_{V,N} \\ \left(\frac{\partial T}{\partial N}\right)_{S,V} &= \left(\frac{\partial \mu}{\partial S}\right)_{V,N} \\ \left(\frac{\partial \mu}{\partial V}\right)_{S,N} &= -\left(\frac{\partial P}{\partial N}\right)_{S,V} \end{aligned}$$

b) With the correct definition of P you find

$$\begin{aligned} T(S,V,N) &= \frac{2}{3Nk_B} E(S,V,N), \\ P(S,V,N) &= \frac{2}{3V} E(S,V,N). \end{aligned}$$