Answers to problem set 7
FYS4130 at UiO, Spring 2012

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7.1

	Fermi	Bose	Distinguishable	Maxwell-Boltzmann
a)	D	B	C	E
b)	A	E	C	C
c)	C	E	F	A

7.2

a) –

b) Phonons and photons do not Bose condense at low temperature: their number is not conserved, and as $T \to 0$ the average occupancy of all the modes goes to zero. A laser has a macroscopic occupation of a single photon frequency; this condensation is not due to low temperature.

7.4

–

7.10

$$
\begin{aligned}
Z_1 &= 1 + Me^{-\beta\epsilon} \\
Z_N &= Z_1^N, \text{ since the particles are not interacting} \\
E(T) = \langle E \rangle = -\frac{\partial}{\partial\beta} \ln Z_N &= N\frac{M\epsilon}{M + e^{\beta\epsilon}} \\
\sigma_E^2 = \langle E^2 \rangle - \langle E \rangle^2 = \frac{\partial^2}{\partial\beta^2} \ln Z_N &= N\frac{M\epsilon^2 e^{\beta\epsilon}}{(M + e^{\beta\epsilon})^2} \\
S(T) = \frac{E(T)}{T} + k_\mathrm{B} \ln Z_N &= \frac{N}{T}\frac{M\epsilon}{M + e^{\beta\epsilon}} + Nk_\mathrm{B} \ln(1 + Me^{-\beta\epsilon}) \\
C(T) = \frac{\partial E(T)}{\partial T} &= \frac{1}{k_\mathrm{B} T^2}\sigma_E^2, \text{ plotted below} \\
S(\infty) - S(0) &= Nk_\mathrm{B} \ln(M + 1)
\end{aligned}
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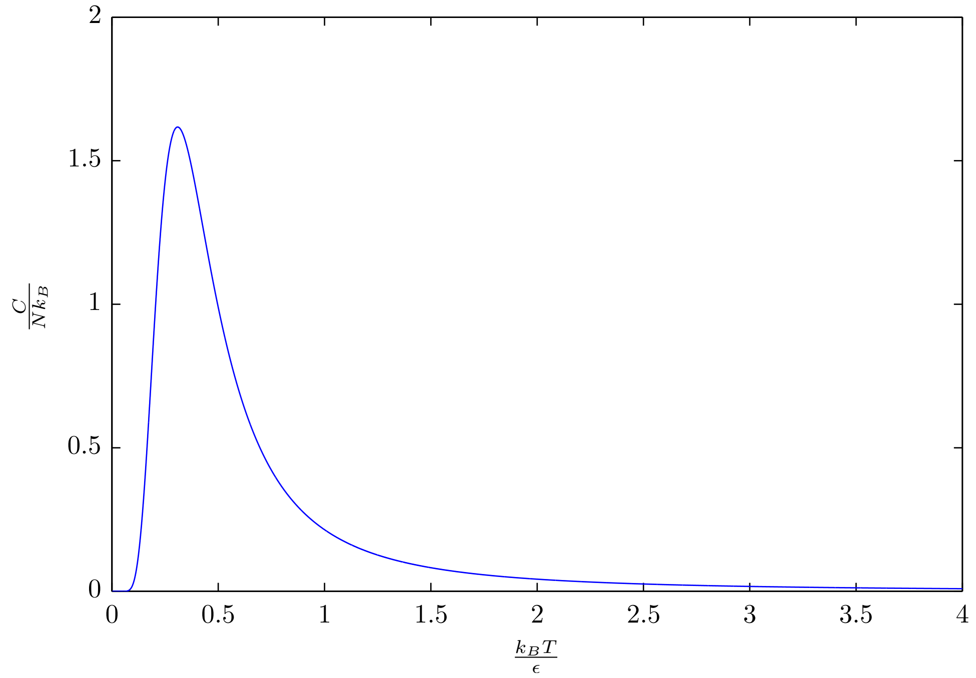

2
1.5
1
0.5
0
$\frac{C}{Nk_B}$
0
0.5
1
1.5
2
2.5
3
3.5
4
$\frac{k_B T}{\epsilon}$