Answers to problem set 8
FYS4130 at UiO, Spring 2012

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7.7

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7.9

a) – (You can ask for written solutions if you need them.)

b) $\frac{M}{V} = \frac{4\Gamma}{\lambda^2 c} = 3.48 \cdot 10^8 \text{ m}^{-3}$.

c) We need $\frac{N+1}{(N+1)+(M-1)} = \frac{1}{2}$.

7.12

a)

$$N + M = \frac{N}{e^{\beta(-\Delta/2-\mu)} + 1} + \frac{M}{e^{\beta(\Delta/2-\epsilon-\mu)} + 1} + \frac{N - M}{e^{\beta(\Delta/2-\mu)} + 1}$$

Since there are $N + M$ particles in total, this equation determines μ.

b) When all states are equally likely, the occupation average for each state is just the number of particles divided by the number of states. The Bose-Einstein distribution also holds, so

$$\frac{N+M}{2N} \approx \frac{1}{e^{-\beta\mu} + 1} \qquad \text{, since } e^{\pm\beta\Delta} \xrightarrow{T\to\infty} 1.$$

$$\mu(T \to \infty) = k_{\mathrm{B}} T \ln\left(\frac{N+M}{N-M}\right)$$

c) At $T = 0$ the electrons fill the energy states from below. With only one particle in each state this fills the valence and impurity bands completely, leaving the conduction band empty.

In the low T limit, excitations from the valence band are extremely unlikely, so we need only consider the impurity band and the conduction band. We find

$$\mu(T \to 0) = \frac{\Delta}{2} - \frac{\epsilon}{2}.$$

d) $\mu(T_{\text{room}}) \approx 0.2$ eV.

The fraction of ionized phosphorous atoms is the fraction of states in the impurity band that are unoccupied, which is very close to one.

The density of holes is the fraction of unoccupied states in the valence band times the number density N/V of states in this band, and is $\rho_{\text{holes}} \approx 2 \cdot 10^9 \text{ cm}^3$.

7.13

You can show by going through section 7.6.3 that the condition for Bose condensation at low temperature can be rewritten in the form d). The others are false or not necessary.

7.16

a)

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\begin{aligned}
E_{\mathrm{el}} &= \frac{1}{120\pi}\frac{\hbar^2}{m_{\mathrm{e}}R^2}\left(\frac{9M\pi}{m_{\mathrm{p}}}\right)^{5/3} = A/R^2 \\
E_{\mathrm{G}} &\overset{\text{uniform density}}{=} -\frac{3}{5}\frac{GM^2}{R} = B/R \\
R_{\min} &= \frac{2A}{B}
\end{aligned}
$$

b) $\epsilon_F = 48$ keV (or a factor of 4 larger?)

i) Typical chemical binding energy is ~ 1 eV $\Rightarrow$ safe to ignore.

ii) $k_{\mathrm{B}}T \approx 1$ keV $\Rightarrow$ safe to ignore.

iii) $m_{\mathrm{e}}c^2 = 512$ keV $\Rightarrow$ cannot be ignored completely.

iv) $(m_{\mathrm{n}} - m_{\mathrm{p}})c^2 = 1.3$ MeV $\Rightarrow$ electrons and protons forming neutrons should be largely suppressed.

c)

$$
\begin{aligned}
E_{\mathrm{el}} &= \frac{V}{\pi^2(\hbar c)^3}\frac{\epsilon_F^4}{4} \\
M_0 &= 1.7M_{\odot}
\end{aligned}
$$