Answers to problem set 12
FYS4130 at UiO, Spring 2012

Jørgen Trømborg

May 2012

11.3

a) –

b) When the vapor pressure hits atmospheric pressure, the water boils, so the true vapor pressure of water at $T = 373\,\mathrm{K}$ is $P_v \approx 10^6$ dynes/cm^2. The caption to Fig. 11.14 says $P_v = 1.5 \cdot 10^7$ dynes/cm^2, which is off by a factor of 15.

c) A lowest-order approximation to the integral in equation (11.23) is

$$\Delta G \approx \rho_{\mathrm{water}} \times \text{barrier height in } \mu \times Ah,$$

where A is an arbitraty area and h is the thickness of the transition zone.

If you're confused by this exercise, try following this argument: Consider an interface between water and vapor that is uniform over a large area A. We want the Gibbs free energy per unit area, so A will cancel out in the end. Under this assumption equation (11.23) is effectively one-dimensional. Starting the integration inside the water, the integrand is zero because $\mu(\rho_{\mathrm{water}}) = \mu_0$. Once we reach the gas, the integrand is again zero because $\mu(\rho_{\mathrm{gas}}) = \mu_0$. In between, there will be a region where ρ changes in some gradual way from ρ_{water} to ρ_{gas}. According to equation (11.22) and the corresponding Fig. 11.14, this raises $\mu(\rho)$: there is an energy barrier to cross. The exact value of the integral will depend on the exact shape $\rho(x)$. Since we don't know this shape, we have to settle for an approximation. An upper bound on the integral is to take the maximum value of ρ, which is $\rho_{\mathrm{water}} \approx 0.025$ mol/cm^3, the maximum value of $\mu(\rho) - \mu_0$, which is $4 \cdot 10^{-14}$ erg/molecule from the figure, and the width of the interval, which is $h = 3$ Å by assumption.

We get $\Delta G/A \approx 18$ dynes/cm, whics is off by a factor of about 3. Not so bad, considering that we used a coarse-grained measure for a very sharp interface.

11.5

1) True

2) False

3) True. Note that Γ_0 and D depend on μ. The rationale for keeping the μ in $\left(\frac{\sigma_{xy}}{\mu}\right)^{D/k_{\mathrm{B}}T}$ is that it keeps the base of the power-law unitless.

4) False. The strain rate depends on the nucleation rate and velocity of dislocations, and so will have a power-law dependence on stress, not a linear dependence.

The behaviour of 2D crystals is subtle, and a fuller understanding of the differences between crystals and liquids in 2D does not come for free. According to Luiza

> “This is an interesting problem, and Sethna’s answer is very much simplified. The thing is that 2d crystals are rather very different than 3d crystals. While melting in 3d is a well-known abrupt phase transition, melting in 2d can happen through a Kosterliz-Thouless phase transition, which is continuous. In short, at low temperature, dislocations are bound in pairs, and as the temperature increases near the melting point (but still slightly below it) thermal fluctuations tend to break dislocation pairs and create a dislocation gas-like phase; this is K-T transition when the crystal (phase with broken translations and rotation symmetries) becomes instead a liquid-crystal (broken rotational symmetry); upon increasing T, the isotropic liquid phase is obtained. The point is that at higher T, and in 2d, crystals and liquids may not be that different in terms of shear deformations. When dislocations are like a free gas, then the strain rate is almost proportional to stress like for a liquid. However, at low T, due to the power law nucleation rate, the strain rate is also nonlinear with stress. The strain rate is here related to the velocity of moving dislocations times their density; but their density is determined by the nucleation rate, thus, the strain rate is proportional to the nucleation rate. That would be the simple argument.”