Answers to problem set 13
FYS4130 at UiO, Spring 2012

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12.4

a) –

b) With $x^* = \pm\sqrt{\mu}$ and $\mu_c = 0$, $\beta = 1/2$.

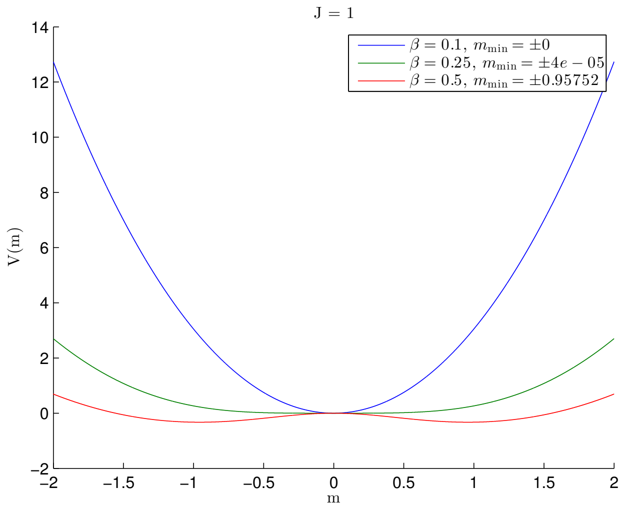
c)

$$\lambda_c = 1$$
$$m \approx \left(\frac{3}{\lambda^3}(\lambda - \lambda_c)\right)^{1/2}$$

12.5

a) $\langle s \rangle = \tanh(4mJ\beta)$, with $\langle \cdot \rangle$ indicating an average over states; the two states are $s = \pm 1$. Self-consistency requires $m = \langle s \rangle$. Using the result from 12.4c) (identify $\lambda = 4J\beta$) we find $T_c = \frac{4J}{k_B}$. The exponent $\beta = 1/2$ (not to be confused with the usage $\beta = \frac{1}{k_B T}$) is in poor agreement with 2D and 3D, but from 4D and up mean field theory works for this system.

b) The free energy $V(m)$ has a single or two degenerate minima, indicative of a continuous phase transition.



c) The free energy $V(m, H)$ has two minima of different energy, indicative of an abrupt phase transition. The metastable state becomes completely unstable when the local minimum of highest energy disappears, approximately at $H = 1$.