# Problem set 3
# FYS4130 at UiO Spring 2014

January 28, 2014

## 1 Harmonic oscillator

Consider a system of localized, independent, 3-dimensional harmonic oscillators. They obey the Hamiltonian

$$H = \sum_{i=1}^{3N} \left( \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 q_i^2 \right).$$

a) Use the phase space volume $\Omega$ and the density of states $\Sigma$ to convince yourself that the number of microstates is

$$W = \frac{1}{(2\pi\hbar)^{3N}} \frac{\pi^{3N}}{(3N)!} \left( \frac{2mE}{m\omega} \right)^{3N} = \frac{1}{(3N)!} \left( \frac{E}{\hbar\omega} \right)^{3N}.$$

b) Show that when $3N \gg 1$ the entropy is

$$S = 3Nk \left( 1 + \log \frac{E}{3N\bar{h}\omega} \right).$$

c) Use the entropy to calculate the internal energy. Does the result agree with the equipartition theorem? *Hint: Calculate $T$ as a function of $E$ and invert this relation to find $E(T)$.*

d) Find the specific heat $C$.

## 2 From the booklet

Do problems 4.19, 4.20 and 4.23 in the booklet. Note that there is a sign error on the right hand side of problem 4.20a, that is, the answer you should get is

$$S - S_0 = k_B \sum_r P_r \ln \frac{P_r^{(0)}}{P_r}.$$