FYS 4130 Statistical Mechanics

Homework 10 April 16, 2010

1) Boson Gas

Consider a gas of spin zero particles of mass m and chemical potential μ in a box of volume V.

a) Calculate the grand canonical partition function. Approximate the sum over states with an integral over energy levels using the density of states:

$D(\epsilon) = \frac{2\pi}{\hbar^3}(2m)^{3/2}\epsilon^{1/2}$

b) Using an integral over the energy to approximate the sum is a good approximation as long as the occupancy of each energy level is small. For a boson gas as you go lower in temperature the occupancy of the lowest energy level is no longer small. Add the contribution to the sum in the partition function from the lowest energy state $\epsilon_0 = 0$.

c) Find the equation of state.

Solution:

a) $\ln \Xi = \frac{V}{\Lambda^3} g_{5/2}(\lambda)$

b) $\ln \Xi = \frac{V}{\Lambda^3} g_{5/2}(\lambda) - \ln(1-\lambda)$

c) $PV = \frac{kTv}{\Lambda^3} g_{5/2}(\lambda) - kT \ln(1-\lambda)$

2) Bose-Einstein condensation

Bose-Einstein condensation is the effect where s $T \to 0$, $\mu \to 0$ and all the particles move into the ground state. The critical temperature T_c is the temperature at which μ approaches zero. Below the critical temperature the occupancy of the ground state becomes large, above the critical temperature the occupancy of the ground state is small and the particles are all in excited states.

a) Use the formula for the total number of particles to calculate the critical temperature.

$T_c = \frac{2\pi\hbar^2}{km}\left(\frac{\rho}{2.612}\right)^{2/3}$

b) For temperatures below the critical temperature, $T < T_c$ find the fraction of particles in excited states N_e/N. Find the fraction of particles in the ground state, N_0/N.

$N_0/N = 1 - (T/T_c)^{3/2}$