FYS 4130 Statistical Mechanics

Homework 8 March 19, 2010

1) 3 Dimensional Debye Solid

In the Debye model for a 3D solid of N atoms, the energy per mode is given by $E = \hbar\omega$. There are $3N$ allowed modes with the highest mode at the Debye frequency ω_D Which is defined by

$\int_0^{\omega_D} d\omega D(\omega) = 3N$

a) Find an expression for the energy and heat capacity.

b) What is the temperature dependence of C_v in the limit T $\rightarrow \infty$?

c) What is the temperature dependence of C_v in the limit T $\rightarrow 0$?

Solution:

For $T \rightarrow \infty$, $U = 3NkT$, $C_v = 3Nk$

For $T \rightarrow 0$, $U = \frac{9N}{(\hbar\omega_D)^3}\frac{\pi^4}{15}4k^4T^3$

2) Zero Point energy

Consider a 3 dimensional isotropic solid of N atoms treated as harmonic oscillators, with the Debye density of states. In addition to the energy in the excitations of the oscillators (the phonons) each oscillator has a zero point energy $\epsilon_0 = \hbar\omega/2$. Find the zero point energy of the system.

$D(\omega) = \frac{3V\omega^2}{2\pi^2 v^3}$

$E_0 = \frac{9}{8}N\hbar\omega_D$

3) Nonrelativistic Fermi Gas

Consider a gas of spin 1/2 fermions, nonrelativistic so that $\epsilon = \frac{p^2}{2m}$.

a) Calculate the fermi energy.

b) Calculate the total energy of the gas at temperature $T = 0$.

c) Calculate the pressure at $T = 0$.

Solution:

a) $\epsilon_f = \frac{\hbar^2}{2m}(3\rho\pi^2)^{2/3}$

b) $E_0 = \frac{3}{5}N\epsilon_f$

c) $P_0 = \frac{\hbar^2}{15m\pi^2}(3\pi^2\rho)^{5/3}$

4) Relativistic Fermi Gas

Consider a gas of spin 1/2 fermions, relativistic so that $\epsilon = pc$ and the density of states including the degeneracy is:

$$g(\epsilon)D(\epsilon) = \frac{V}{\pi^2\hbar^3c^3}\epsilon^2$$

a) Calculate the fermi energy.

b) Calculate the ground state energy, Which is the total energy of the gas at temperature $T = 0$.

c) Calculate the pressure at $T = 0$.

Solution:

a) $\epsilon_f = (3\rho\pi^2)^{1/3}\hbar c$

b) $E_0 = v\rho\frac{3}{4}(3\rho\pi^2)^{1/3}\hbar c$

c) $P_0 = \frac{\hbar c}{4}(3\pi^2)^{1/3}\rho^{4/3}$